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Superstrings on AdS_3 and symmetric products

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Abstract: Some properties of $N = 2$ superstrings on $AdS_3 \times \mathcal{N}$ are studied. Their spectrum has the same pattern as 2-d CFTs in the moduli space of symmetric products $\mathcal{M}^p/S_p$, where p is associated with the number of long fundamental strings required to construct the AdS_3 background, and $\mathcal{M}$ is the spacetime CFT corresponding to short string excitations of a single long string. Worldsheet operators associated with w long strings correspond to twisted sectors of $\mathcal{M}^p/S_p$.

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1. Introduction and summary

In this paper we continue the study of the superstring on $AdS_3 \times \mathcal{N}$. We use the NSR formulation; for earlier works see, e.g., [1]–[13]. The study in this work leads us to conjecture that the spacetime theory is a two dimensional CFT in the moduli space of the symmetric product:

$$(\mathcal{M}_{6k})^p / S_p \,. \tag{1.1}$$

Here k is the level of the $AdS_3 \simeq \mathrm{SL}(2)$ worldsheet CFT. $\mathcal{M}_{6k}$ is a two dimensional CFT with a central charge $c = 6k$, the precise structure of which depends on the details of the internal worldsheet CFT $\mathcal{N}$. The integer number p, which parametrizes the string coupling g_s, is associated with the number of fundamental "long" strings required to construct the AdS_3 background. More precisely, it has one of the following (related) interpretations:

(a) To leading order in $1/p$ [2]:

$$g_s^2 = \frac{V_{\mathcal{N}}}{4p\sqrt{k}} \,, \tag{1.2}$$

where $V_{\mathcal{N}}$ is the volume of the compactification manifold $\mathcal{N}$ (measured in string units).

(b) p is the number of fundamental strings used to generate the AdS_3 background [10]: starting with the background $R^{1,1} \times R_\phi \times \mathcal{N}$, where R_ϕ is the real line with a linear dilaton, one obtains the $AdS_3 \times \mathcal{N}$ background by placing p fundamental strings stretched in $R^{1,1}$ and then going to their near horizon limit.

(c) p is the maximal number of "long strings" [2, 6, 7, 11, 14], which can be emitted in the AdS_3 background and stretch in its boundary.

(d) The "exclusion principle" [15] allows at most "p-particle" (BPS) states.

The spacetime Hilbert space of $\mathcal{M}_{6k}$ consists of the "single particle" states corresponding to worldsheet vertex operators in the "short string" sector, and the continuum of a single "long string."

For concreteness of the discussion, we shall consider the family of supersymmetric string backgrounds which lead to at least $N = 2$ supersymmetry in spacetime [8, 9]. In particular, we will verify that the pattern of the spacetime chiral spectrum of these theories is in agreement with the picture conjectured above.

To do that, we begin in section 2 by reviewing the basic properties of a bosonic string propagating in AdS_3, and introduce a "twist" required to impose single valuedness of the wave functions on AdS_3 "intrinsically" in the AdS_3 worldsheet theory. This leads to the results of [11], which were obtained using spectral flow arguments. In particular, the sector of w long strings in [11] is associated with the w'th twisted sector.

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In section 3, we review the $N = 2$ superstring on $AdS_3 \times \mathcal{N}$ and introduce the "twist" leading to the analogue of [11] in the case of a superstring. In section 4, we study observables leading to chiral "single particle states" of the spacetime $N = 2$ SCFT. In section 5, we show that the pattern of this chiral spectrum coincides with the one of (1.1). In particular, the chiral operators associated to N long strings correspond to operators in the Z_N twisted sector of $(M_{6k})^p/S_p$. In section 6, we present some examples, and in section 7 we discuss a few points. Finally, some of the technical details appear in the appendices A and B.

A related work on string theory on $AdS_3 \times \mathcal{N}$ using discrete light-cone Liouville theory methods appeared in [13].

2. The Bosonic string on AdS_3

In this section we review some basic facts about bosonic string propagation on AdS_3 space [1, 2, 6]. We then show how properly taking into account single valuedness of the wave function forces one to include new sets of operators which were argued to exist in [11], where reasoning based on spectral flow was used.

2.1 Review

String theory on an $AdS_3 \times \mathcal{N}$ background is formulated on the worldsheet as the product of an SL(2) WZW model and the CFT on $\mathcal{N}$.[1] $AdS_3 \simeq$ SL(2) has three SL(2) affine currents $J^a(z)$ and anti-currents $\bar{J}^a(\bar{z})$, $a = \pm, 3$, generating an affine $\mathrm{SL}(2)_L \times \mathrm{SL}(2)_R$ algebra at level k. For simplicity of the discussion we will many times refer only to left-movers. As usual in string theory, the spacetime theory has three conserved charges:

$$\begin{aligned} L_0 &= -\oint dz J^3(z)\,, \\ L_1 &= -\oint dz J^+(z)\,, \\ L_{-1} &= -\oint dz J^-(z)\,, \end{aligned} \tag{2.1}$$

which satisfy the SL(2) Lie algebra $[L_n, L_m] = (n-m)L_{n+m}$, $n, m = 0, \pm 1$. A special property of string theory on AdS_3 is that this Lie algebra is enhanced to the full Virasoro algebra:

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c_{st}}{12}(n^3 - n)\delta_{n+m,0}\,, \qquad n \in \mathbb{Z}\,. \tag{2.2}$$

[1]By SL(2) we mean the universal cover of the SL(2) group manifold, so that the time-like direction is non-compact

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The construction of the worldsheet vertex operators corresponding to the spacetime generators L_n is given in [2, 6]. For future purpose, we pick up the presentation of L_n in the free field realization of SL(2):

$$L_n = \oint dz \left[(n^2-1)J^3\gamma^n - \frac{n(n-1)}{2}J^-\gamma^{n+1} - \frac{n(n+1)}{2}J^+\gamma^{n-1} \right] . \tag{2.3}$$

Here γ is the dimension zero field in the free field representation of SL(2); for the details of the construction (2.3) see [2]. The spacetime central charge c_{st} is related to the worldsheet SL(2) level k via:

$$c_{st} = 6kp\,, \tag{2.4}$$

where p is the integer number discussed in the introduction (1).

Let us introduce a canonically normalized scalar X: $X(z)X(z') \sim -\log(z-z')$, in terms of which we write:

$$J^3 = -\sqrt{\frac{k}{2}}\partial X\,. \tag{2.5}$$

We can now decompose any operator Φ in AdS_3 to $\mathrm{SL(2)}/\,\mathrm{U(1)} \times \mathrm{U(1)}_X$, where the $\mathrm{U(1)}_X$ is the one generated by J^3; the "parafermionic" SL(2)/ U(1) part of Φ will be denoted by Ψ. In particular, the charged currents decompose as:

$$J^\pm = \Psi_\pm e^{\pm\sqrt{2/k}X}\,. \tag{2.6}$$

The bosonic string on $AdS_3 \times \mathcal{N}$ has physical vertex operators $V(j;m,\bar{m})$ which are labeled, in particular, by their SL(2) numbers j, m, $\bar{m}$, where j is related to the second Casimir of SL(2), $-j(j+1)$, and $m, \bar{m}$ are the eigenvalues of $J^3, \bar{J}^3$, respectively. $V(j;m,\bar{m})$ includes the contribution of a function $\Phi_{jm\bar{m}}$ on AdS_3 which decomposes as:

$$\Phi_{jm\bar{m}} = \Psi_{jm\bar{m}} e^{\sqrt{2/k}[mX(z)+\bar{m}\bar{X}(\bar{z})]}\,, \tag{2.7}$$

where the SL(2)/ U(1) part has left-handed scaling dimension

$$\Delta(\Psi_{jm\bar{m}}) = \frac{-j(j+1)}{k-2} + \frac{m^2}{k}\,, \tag{2.8}$$

and similarly for the right-moving scaling dimension, so that:

$$\Delta(\Phi_{jm\bar{m}}) = \frac{-j(j+1)}{k-2}\,, \tag{2.9}$$

as it should. The operators $\Phi_{jm\bar{m}}$ are primaries of $\widehat{\mathrm{SL}(2)}_L \times \widehat{\mathrm{SL}(2)}_R$. Single valuedness of the wave function on AdS_3 leads to the constraint:

$$m - \bar{m} \in \mathbb{Z}\,. \tag{2.10}$$

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We will shortly discuss how to implement this condition.

Next we discuss to which SL(2) representations the operators $\Phi_{jm\bar{m}}$ belong. Unitarity of the SL(2)/ U(1) theory implies that j belongs to one of the following regions [16] (for early work on unitarity of SL(2)/ U(1) see [1]):

$$j \in \mathbb{R}, \qquad -\frac{1}{2} < j < \frac{k-3}{2}, \qquad \Delta(\Psi_{jm\bar{m}}) \geq 0, \tag{2.11}$$

or

$$j \in -\frac{1}{2} + i\mathbb{R}, \tag{2.12}$$

where $\Delta(\Psi_{jm\bar{m}})$ is given in (2.8). When the SL(2)/ U(1) $\times S^1_X$ is recomposed into an SL(2), operators with j in the ranges above fall into three kinds of representations of SL(2) [11] (see [1] for earlier works). Operators in the range (2.11) fall into either lowest weight principal discrete representations:

$$\mathcal{D}^+_j: \qquad m = j + n, \qquad n = 1, 2, \ldots \tag{2.13}$$

or highest weight principal discrete representations:

$$\mathcal{D}^-_j: \qquad m = -(j + n), \qquad n = 1, 2, \ldots \tag{2.14}$$

Operators with j in the range (2.12) fall into principal continuous representations of SL(2):

$$\mathcal{C}_{j,\mu}: \qquad m = \mu, \mu \pm n, \qquad 0 \leq \mu < 1, \qquad n = 1, 2, \ldots \tag{2.15}$$

Let us now comment on the spacetime interpretation of vertex operators $V(j; m, \bar{m})$, which are primaries of $\widehat{\mathrm{SL}(2)}$, built from the various SL(2) representations above. The principal continuous representations $\mathcal{C}_{j,\mu}$ are associated with tachyonic states. On the other hand, local observables correspond to vertex operators $V(j; m, \bar{m})$ in the highest weight representations $\mathcal{D}^-_j$ and lowest weight ones $\mathcal{D}^+_j$ of SL(2). They correspond in spacetime to the modes $A^h_{m\bar{m}}$ of a primary field with weight $h = j + 1$. To see that, recall that the commutator of a physical vertex operator with the spacetime Virasoro generators is given by [2]:

$$[L_n, V(j; m, \bar{m})] = (nj - m)V(j; m + n, \bar{m}), \tag{2.16}$$

while in a two dimensional CFT:

$$[L_n, A^h_{m\bar{m}}] = [n(h - 1) - m]A^h_{m+n,\bar{m}}. \tag{2.17}$$

In particular, the operators in $\mathcal{D}^-_j$ are the creation operators, generating ingoing states from the spacetime vacuum, while those in $\mathcal{D}^+_j$ are their hermitian conjugates.

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2.2 The "twist field" and the new operators

Above we remarked that single valuedness of the wave function on AdS_3 space requires (2.10). In [2, 6] the condition (2.10) was imposed "by hand." To obtain this condition "intrinsically" in the AdS_3 worldsheet theory, we need to include "twist fields" $t^w_{sl(2)}$ which impose such a condition via mutual locality.

Let us stress that we are not twisting an otherwise consistent theory, rather it is consistency of the theory (tree level unitarity and higher loop modular invariance) which requires the inclusion of these twist fields, and of all the twisted operators which arise through OPEs with the twist fields.

We thus introduce the operator:

$$t^w_{sl(2)}(z,\bar z) = t^w_{sl(2)}\bar t^w_{sl(2)} = e^{-w[\int J^3(z)+\int \bar J^3(\bar z)]} = e^{w\sqrt{\frac{k}{2}}[X(z)+\bar X(\bar z)]}\,, \qquad w\in\mathbb{Z}\,. \tag{2.18}$$

The OPE of $t^w_{sl(2)}$ with $\Phi_{jm\bar m}$ takes the form

$$\begin{aligned} t^w_{sl(2)}(z,\bar z)\Phi_{jm\bar m}(z',\bar z') &\sim (z-z')^{-wm}(\bar z-\bar z')^{-w\bar m}\Phi^w_{jm\bar m}(z',\bar z') \\ &= (z-z')^{-(m-\bar m)w}|z-z'|^{-2\bar m w}\Phi^w_{jm\bar m}\,, \end{aligned} \tag{2.19}$$

where

$$\Phi^w_{jm\bar m} = \Psi_{jm\bar m}e^{\sqrt{\frac{2}{k}}[(m+\frac{k}{2}w)X+(\bar m+\frac{k}{2}w)\bar X]}\,. \tag{2.20}$$

From (2.19) we see that mutual locality with $t^w_{sl(2)}$ indeed implies the condition (2.10). Now, consistency of the theory implies that we must include also the "twisted" operators $\Phi^w_{jm\bar m}$ (2.20).

The scaling dimension of $\Phi^w_{jm\bar m}$ is:

$$\Delta(\Phi^w_{jm\bar m}) = \Delta(\Psi_{jm\bar m}) - \frac{[m+(k/2)w]^2}{k} = \frac{-j(j+1)}{k-2} - \frac{k}{4}w^2 - mw\,. \tag{2.21}$$

Considering now a vertex operator V^w defined by:

$$V^w = V_\Delta \Phi^w_{jm\bar m}\,, \tag{2.22}$$

where V_Δ is a primary operator in the CFT on $\mathcal{N}$ with weight Δ, the physicality condition is:

$$-\frac{j(j+1)}{k-2} - \frac{k}{4}w^2 - mw + \Delta = 1\,. \tag{2.23}$$

To find the spacetime properties of V^w we first need to recall the following OPE (see [2] for details):

$$J^3(z)\gamma^n(z') \sim \frac{n\gamma^n(z')}{z-z'}\,. \tag{2.24}$$

This and (2.5) imply that:[2]

$$\gamma = \Psi_\gamma e^{\sqrt{2/k}\,X}\,, \tag{2.25}$$

[2] γ^m is the free field representation of Φ_{0m} (see [2]).

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and therefore, using (2.20) and the fact that γ^n has a regular OPE with Φ_{jm} (2.7), we find that

$$\gamma^n(z)\Phi^w_{jm}(z') \sim (z-z')^{-nw}O(z'), \tag{2.26}$$

where O is some operator. Second, using (2.5), (2.6) and (2.20), and the standard OPEs in the parafermionic theory (in its free field representation), we find:

$$\begin{aligned} J^3(z)\Phi^w_{jm}(z') &\sim \frac{[m+(k/2)w]\Phi^w_{jm}(z')}{z-z'}, \\ J^\pm(z)\Phi^w_{jm}(z') &\sim (z-z')^{\mp w-1}(m \mp j)\Phi^w_{j,m\pm 1}(z') + \cdots . \end{aligned} \tag{2.27}$$

Hence, from (2.3), (2.26) and (2.27), we find that for $w<0$:

$$\begin{aligned} [L_0, V^w] &= -\left(m + \frac{k}{2}w\right) V^w \\ [L_n, V^w] &= 0\,, \qquad n \geq 1\,. \end{aligned} \tag{2.28}$$

From (2.28), one learns that the spacetime state $|V^w\rangle$ created by V^w from the vacuum obeys:

$$L_0|V^w\rangle = h|V^w\rangle\,, \qquad L_n|V^w\rangle = 0\,, \qquad n \geq 1\,, \tag{2.29}$$

where

$$h = -\left(m + \frac{k}{2}w\right). \tag{2.30}$$

Hence $|V^w\rangle$ is a primary state with weight h. Since V^w obeys (2.28), rather than (2.16), it corresponds in spacetime to the mode $m_{st} = -h$ of a primary field plus something which annihilates the spacetime vacuum. The latter piece is presumably an indication of the non-locality of the $w \neq 0$ sectors,[3] due to its relation to long strings [11]. The part of V^w corresponding to a mode of a primary, on which we focus, is however a local observable.

Solving (2.23) for m in (2.30), we get:

$$h = \frac{k|w|}{4} + \frac{1}{|w|}\left(-\frac{j(j+1)}{k-2} + \Delta - 1\right), \tag{2.31}$$

in agreement with [11].

A few comments are in order:

1. Equation (2.28) implies that all operators with $w<0$ correspond to creation operators, up to something which annihilates the vacuum. Namely, also operators Φ_{jm} in the $\mathcal{D}^+_j$ representation, which correspond to annihilation operators, when twisted with $w<0$ create incoming states from the spacetime vacuum. This is not a surprise since in the spectral flow picture [11], the $w=-1$ flow of the $\hat{\mathcal{D}}^+_j$ representation of $\widehat{\mathrm{SL}(2)}$ is nothing else than $\hat{\mathcal{D}}^-_{\tilde{j}=\frac{k}{2}-j-2}$.

[3]In local CFTs operators cannot obey (2.28). However, we expect that the spacetime CFT corresponding to string theory on AdS_3 with a vanishing RR background is a non-local (singular) CFT [7].

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2. For $w = 0$, the spacetime primary states are created from the vacuum by the operators corresponding to $\Phi_{j,-j-1}$, which is the highest weight of $\mathcal{D}_j^-$. On the other hand, (2.28) implies that Φ^w_{jm} for any m (compatible with (2.13) or (2.14)) create from the vacuum a primary state. This again can be understood using the spectral flow picture, where one finds that all the primary states of $\widehat{\mathrm{SL}(2)}$, which sit in a $\mathcal{D}_j^\pm$ representation of SL(2), flow to states in the representation $\hat{\mathcal{D}}_j^{\pm,w}$ (in the notation of [11]) that are all highest weights of SL(2) (note that $w < 0$). The other states in this SL(2) representation correspond to twists of $\widehat{\mathrm{SL}(2)}$ descendants.

3. The above discussion can be repeated for $w > 0$, exchanging ingoing for outgoing states. This is compatible with the picture where w is related to a (non-preserved) winding number of long strings near the boundary of AdS_3, since winding of opposite sign signals charge conjugation.

3. The superstring on AdS_3

Let us first recall the new ingredients involved in a fermionic string on AdS_3 relative to the bosonic case. This theory has affine SL(2) *super*currents

$$\psi^a + \theta\sqrt{\frac{2}{k}}J^a\,, \qquad a = 1, 2, 3\,, \tag{3.1}$$

where

$$J^a = j^a - \frac{i}{2}\epsilon^a{}_{bc}\psi^b\psi^c\,, \tag{3.2}$$

and

$$\begin{aligned} \eta^{ab} &= \mathrm{diag}(+,+,-), \\ \psi^a(z)\psi^b(z') &\sim \frac{\eta^{ab}}{z-z'}\,, \\ J^a(z)J^b(z') &\sim \frac{(k/2)\eta^{ab}}{(z-z')^2} + \frac{i\epsilon^{ab}{}_cJ^c}{z-z'}\,. \end{aligned} \tag{3.3}$$

The purely bosonic currents j^a generate an affine SL(2) algebra at level $k+2$ and commute with ψ^a, whereas the total (physical) currents J^a generate a level k SL(2) algebra[4] and act on ψ as follows from (3.2), (3.3). Therefore, the "untwisted" operators $\Phi_{jm\bar{m}}$ have scaling dimension:

$$\Delta(\Phi_{jm\bar{m}}) = \frac{-j(j+1)}{k}\,. \tag{3.4}$$

The unitarity bound on j, written in terms of the total level, is now [11, 16]:

$$-\frac{1}{2} < j < \frac{k-1}{2}\,. \tag{3.5}$$

[4]From now on we denote by k the total level of SL(2), while in the previous section it denoted the bosonic level.

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3.1 Review of the $N = 2$ superstring on $AdS_3 \times \mathcal{N}$

To construct a *superstring* on $AdS_3 \times \mathcal{N}$ we need to impose a chiral GSO projection. In the following we review the construction of [8], [9] leading to $N = 2$ supersymmetry in the boundary two dimensional CFT (the spacetime theory). There are three conditions on the CFT background $\mathcal{N}$:

1. $\mathcal{N}$ is a SCFT with central charge

$$c_{\mathcal{N}} = 15 - c_{sl(2)} = 15 - \left(\frac{9}{2} + \frac{6}{k}\right) = \frac{21}{2} - \frac{6}{k}\,, \tag{3.6}$$

   so that the total worldsheet central charge on $AdS_3 \times \mathcal{N}$ is critical: $c = 15$.

2. $\mathcal{N}$ has an affine U(1) which we write in terms of a canonically normalized scalar Y as:

$$J^Y = i\partial Y\,; \tag{3.7}$$

   we denote its worldsheet fermionic superpartner by χ^Y.

3. The quotient CFT $\mathcal{N}/\,\mathrm{U}(1)$, where the U(1) we gauge is with respect to J^Y (3.7), has an $N = 2$ supersymmetry. Let $J_R^{\mathcal{N}/\,\mathrm{U}(1)}$ be the $\mathrm{U}(1)_R$ current of the $N = 2$ worldsheet superconformal algebra of $\mathcal{N}/\,\mathrm{U}(1)$. We can write it in terms of a canonically normalized scalar Z as:

$$J_R^{\mathcal{N}/\,\mathrm{U}(1)} = i\sqrt{\frac{c_{\mathcal{N}/\,\mathrm{U}(1)}}{3}}\partial Z = i\sqrt{3 - \frac{2}{k}}\partial Z\,. \tag{3.8}$$

We introduce two additional canonically normalized scalars H_1, H_2, the bosonizations of the $\mathrm{SL}(2) \times \mathrm{U}(1)_Y$ fermions:

$$\partial H_1 = \psi^1\psi^2\,, \qquad i\partial H_2 = \psi^3\chi^Y\,. \tag{3.9}$$

The *spacetime* supercharges are constructed as [17]:

$$G_r^{\pm} = (2k)^{1/4}\oint dz\, e^{-\varphi/2}S_r^{\pm}\,, \qquad r = \pm\frac{1}{2}\,, \tag{3.10}$$

where φ is the scalar field arising in the bosonized superghost system of the worldsheet supersymmetry, and the spin fields $S_r^{\pm}$ are (see appendix A for the details of the computation):

$$\begin{aligned} S_r^+ &= e^{-ir(H_1+H_2)-\frac{i}{2}\sqrt{3-\frac{2}{k}}Z+i\sqrt{\frac{1}{2k}}Y}\,, \\ S_r^- &= e^{-ir(H_1-H_2)+\frac{i}{2}\sqrt{3-\frac{2}{k}}Z-i\sqrt{\frac{1}{2k}}Y}\,. \end{aligned} \tag{3.11}$$

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The algebra generated by the supercharges is

$$\begin{aligned}
\{G_r^+, G_s^-\} &= 2L_{r+s} + (r-s)J_0\,, \qquad & r,s = \pm\frac{1}{2} \\
L_m, L_n &= (m-n)L_{m+n}\,, & m,n = 0,\pm 1 \\
L_m, G_r^\pm &= \left(\frac{m}{2} - r\right) G^\pm_{m+r}[J_0, G_r^\pm] = \pm G_r^\pm
\end{aligned} \tag{3.12}$$

with all other (anti-)commutators vanishing. Up to picture-changing [17], $L_0, L_{\pm 1}$ are given by (2.1) (with J^a being the total currents), while J_0 is given by:

$$J_0 = \oint dz J^R(z) = \sqrt{2k}\oint dz J^Y(z). \tag{3.13}$$

The algebra (3.12) is a global spacetime $N = 2$ superconformal algebra whose $\mathrm{U}(1)_R$ generator is J_0.[5]

Commuting L_n, the generators of the full spacetime Virasoro algebra (2.2), with the generators of the global algebra (3.12) gives a full spacetime $N = 2$ superconformal algebra in the spacetime NS sector with modes $G_r^\pm$, $r \in \mathbb{Z} + 1/2$ and J_n, $n \in \mathbb{Z}$. Physical states are constructed using physical vertex operators that are local with respect to the supercharges (3.10); this is the analogue of the usual GSO projection.

3.2 The supersymmetric "twist field"

To obtain a consistent superstring theory on AdS_3 we need to include a twist operator which imposes the single valuedness condition (2.10). The twist field $t^w_{sl(2)}$ in (2.18), with J^3 being the *total* current, is not good because it is not mutually local with the spacetime supercharges $G_r^\pm$ (3.10) and (3.11).[6] A good twist operator t^w which is mutually local with the supercharges acts also on the $\mathrm{U}(1)_Y$ factor of $\mathcal{N} \simeq \mathcal{N}/\,\mathrm{U}(1) \times \mathrm{U}(1)_Y$. Explicitly, we include in the superstring the operator:[7]

$$t^w = t^w_{sl(2)} t^w_{u(1)} = e^{-w\int J^3} e^{w/2\int J^R} = e^{w\sqrt{k/2}(X+iY)}\,, \tag{3.14}$$

where J^R is given in (3.7) and (3.13).

Let us check that indeed t^w is mutually local with the spacetime supercharges. To do that, we use the facts:

$$J^3 = j^3 - i\partial H_1\,, \tag{3.15}$$

where j^3 is the bosonic current and H_1 is given in (3.9), hence,

$$e^{-\int J^3} \sim e^{iH_1}\,, \tag{3.16}$$

[5]Here we took $J_0 \to -J_0$ and $S^+ \leftrightarrow S^-$ relative to [8], to be compatible with the conventions of [10] used later.

[6]We could have used the bosonic twist field $e^{-w\int j^3}$ which is mutually local with $G_r^\pm$, but we find it natural to use the total J^3 since the spacetime Virasoro generators (2.3) are constructed [2] from the total SL(2) currents.

[7]From now on, for simplicity, we only write the left moving part of the operators.

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and

$$e^{1/2\int J^R} = e^{i\sqrt{k/2}Y}\,. \tag{3.17}$$

Using (3.14), (3.16) and (3.17), and the form of the spin fields $S_r^\pm$ in (3.11) we find that

$$t^w(z)S_r^\pm(z') \sim (z-z')^{-(r\mp1/2)w}O(z')\,, \tag{3.18}$$

where O is some operator. Since $r=\pm1/2$ we get $r\mp1/2\in\mathbb{Z}$ and, therefore, $G_r^\pm$ survive the twist.

Comments:

1. The combinations $J^3\pm1/2J^R$ are null and thus, in particular,

$$\Delta(t^w)=0\,. \tag{3.19}$$

2. As discussed before, we may decompose the $AdS_3\times\mathcal{N}$ CFT on $AdS_3\times S^1_Y\times\mathcal{N}/\,\mathrm{U}(1)$. Consistency with the GSO projection (3.11) requires S^1_Y to be a circle of radius $R_Y=1/\sqrt{k}$ [16] (we work in the normalization where the self-dual radius, obtained at $k=1$ [16], is 1). This means, in particular, that in the NS-NS sector an exponential factor $e^{i(qY+\bar{q}\bar{Y})}$ in vertex operators has $q=1/\sqrt{2}(P/R_Y+WR_Y)$ and $\bar{q}=1/\sqrt{2}(P/R_Y-WR_Y)$, where $P,W\in\mathbb{Z}$. Invariance under the twist imposes the condition $(m-\bar{m})+\sqrt{k/2}(q-\bar{q})\in\mathbb{Z}$. But in the NS-NS sector we have $\sqrt{k/2}(q-\bar{q})=\sqrt{k}WR_Y=W\in\mathbb{Z}$, and therefore in the NS-NS sector only operators obeying $m-\bar{m}\in\mathbb{Z}$ survive the twist. In the R-NS sector, (3.11) implies that $q\to q\pm\sqrt{1/2k}$, therefore, $\sqrt{k/2}(q-\bar{q})=W\pm1/2\in\mathbb{Z}+1/2$, which further implies $m-\bar{m}\in\mathbb{Z}+1/2$. This is consistent with the expectation that worldsheet operators in the R-NS sector correspond to spacetime fermions.

We chose to introduce in the theory the holomorphic twist operators

$$t^w_+ = e^{w\sqrt{k/2}(X+iY)}\,, \tag{3.20}$$

where $J^3=-\sqrt{k/2}\partial X$. However, one could also choose to twist with their complex conjugates

$$t^w_- = e^{w\sqrt{k/2}(X-iY)}\,. \tag{3.21}$$

The two are related by the automorphism $a_\pm = e^{\pm i\sqrt{2k}Y}$ (which is mutually local with the supercharges (3.10) and (3.11), and it changes the quantized momenta P on S^1_Y by an even number). We will see that there is no ambiguity in considering both of the above types of twist fields.

4. Chiral spectrum

A property of two dimensional theories with at least $N = 2$ supersymmetry is the existence of chiral states, which belong to short supermultiplets. The spectrum of these operators does not change under continuous deformations, since their weight is given in terms of their R-charge. We now turn to study the chiral spectrum of the $N = 2$ spacetime theory, as given by the superstring theory construction of the preceding section. The details of the computations needed to obtain the results of this section are contained in appendix A.

4.1 The untwisted sector

Starting from the NS sector of the worldsheet theory, there are two kinds of vertex operators leading to spacetime chiral states. We first consider the following vertex operators:

$$\mathcal{X}_R = e^{-\varphi} e^{iqY} V \Phi_{jm} \,, \tag{4.1}$$

where V is an operator in the $N = 2$ SCFT on $\mathcal{N}/\,\mathrm{U}(1)$ with weight Δ_V and R-charge equal to r_V. To simplify the discussion, we restrict to $r_V \geq 0$. The inclusion of the $r_V \leq 0$ states amounts to complex conjugation of the spacetime operators.

The operators (4.1) have to satisfy the on-shell condition:

$$-\frac{j(j+1)}{k} + \frac{1}{2}q^2 + \Delta_V = \frac{1}{2} \,, \tag{4.2}$$

and the GSO projection:

$$q\sqrt{\frac{2}{k}} - r_V \in 2\mathbb{Z} + 1 \,. \tag{4.3}$$

The operator (4.1) has spacetime weight $h = j+1$ and spacetime R-charge $R = \sqrt{2k}q$, thus requiring chirality (or anti-chirality) in spacetime implies:

$$j + 1 = h = \pm\frac{R}{2} = \pm\sqrt{\frac{k}{2}}q \,. \tag{4.4}$$

Putting together (4.4), (4.2) and (4.3) (see appendix A for the details), we get the spacetime anti-chiral operators corresponding to the following vertex operators [10]:

$$\mathcal{X}_{-k(1-r_V)} = e^{-\varphi} e^{-i\sqrt{\frac{2}{k}}(j+1)Y} V \Phi_{j=\frac{k}{2}(1-r_V)-1,m} \,, \qquad r_V + \frac{1}{k} - 1 < 0 \,, \tag{4.5}$$

where the inequality is implied by (3.5), and V has to be a chiral operator on the worldsheet, $\Delta_V = r_V/2$.

The second set of operators is:

$$\mathcal{W}_R = e^{-\varphi} e^{iqY} V (\psi\Phi_j)_{j-1,m} \,, \tag{4.6}$$

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where $(\psi\Phi_j)_{j-1}$ is the combination of Φ_j with the SL(2) fermions that has SL(2) spin $j-1$ (see [4] and appendix A for its explicit form). These operators have to satisfy the on-shell condition:

$$-\frac{j(j+1)}{k}+\frac{1}{2}q^2+\Delta_V=0\,, \tag{4.7}$$

and the GSO projection:

$$q\sqrt{\frac{2}{k}}-r_V\in 2\mathbb{Z}\,. \tag{4.8}$$

Now the spacetime weight is $h=j$ and thus the condition for spacetime anti-chirality is $j=\pm\sqrt{k/2}q$. Together with (4.7) and (4.8), we get the following spacetime chiral operator:

$$\mathcal{W}_{kr_V}=e^{-\varphi}e^{i\sqrt{\frac{2}{k}}jY}V(\psi\Phi_{j=\frac{k}{2}r_V})_{j-1,m}\,,\qquad r_V+\frac{1}{k}-1<0\,. \tag{4.9}$$

Again, V is a chiral operator on the worldsheet ($r_V\geq 0$).

Moving to the Ramond sector of the theory, we find one additional class of physical vertex operators which lead to chiral states in spacetime. A general Ramond sector vertex operator will involve dressing operators with spin fields similar to (3.11). Note that the latter will not only change the SL(2) representation of Φ_j to $j\pm 1/2$, but also alter the total charge with respect to both J^Y and $J_R^{\mathcal{N}/\mathrm{U}(1)}$, through their dependence on Y and Z, respectively. We thus consider the following operators:

$$\mathcal{Y}_R=e^{-\frac{\varphi}{2}}(Se^{iqY}V\Phi_j)^{q+\frac{1}{\sqrt{2k}}}_{j-1/2,m}\,. \tag{4.10}$$

The combination above is expanded in appendix A, where the derivation of the following is also given. The spin field in (4.10) is given by:[8]

$$S_{\pm\frac{1}{2}}=e^{\mp\frac{i}{2}(H_1-H_2)-\frac{i}{2}\sqrt{3-\frac{2}{k}}Z+i\frac{1}{\sqrt{2k}}Y}\,, \tag{4.11}$$

the subscript being their J^3 eigenvalue. The operators (4.10) satisfy the on-shell condition:

$$-\frac{j(j+1)}{k}+\frac{1}{2}q^2+\frac{q}{\sqrt{2k}}+\Delta_V-\frac{r_V}{2}=0\,, \tag{4.12}$$

and the GSO projection:

$$q\sqrt{\frac{2}{k}}-r_V\in 2\mathbb{Z}+1\,. \tag{4.13}$$

To derive (4.12) it is useful to write $V=ve^{i\frac{r_V}{a}Z}$, where $a=\sqrt{3-2/k}$ and v has a regular OPE with $J_R^{\mathcal{N}/\mathrm{U}(1)}$. In addition, they are required to be BRST invariant.

The spacetime weight of (4.10) is $h=j+1/2$ and their R-charge is given by $R=\sqrt{2k}q+1$. Requiring (anti-)chirality in spacetime $j+1/2=\pm(\sqrt{k/2}q+1/2)$, we

[8]Note that $e^{-\varphi/2}S_{\pm 1/2}$ on their own are not mutually local with the supercharges (3.11).

end up with the following BRST invariant vertex operator (which leads to a chiral state in spacetime):

$$\mathcal{Y}_{1+k(r_V-1)} = e^{-\frac{\varphi}{2}}(Se^{i\sqrt{\frac{2}{k}}jY} V\Phi_{j=\frac{k}{2}(r_V-1)})_{j-1/2,m} \,. \tag{4.14}$$

As before, V is chiral on the worldsheet with $r_V \geq 0$. The bound (3.5) implies in this case:

$$1 < r_V + \frac{1}{k} < 2 \,. \tag{4.15}$$

Comments:

1. In appendix A it is shown that one can also find BRST invariant vertex operators in the Ramond sector which are anti-chiral in spacetime:

$$\tilde{\mathcal{Y}}_{1+k(r_V-2)} = e^{-\frac{\varphi}{2}} \left(S^+ e^{-i\sqrt{\frac{2}{k}}(j+1)Y} V\Phi_{j=-1+\frac{k}{2}(2-r_V)}\right)_{j-1/2,m} \,, \tag{4.16}$$

   where S^+ is given in (3.11) and the non-negative r_V satisfies the same bounds (4.15). However one can rearrange the Y and Z exponents to get:

$$\tilde{\mathcal{Y}}_{1+k(r_V-2)} = e^{-\frac{\varphi}{2}} \left(S' e^{-i\sqrt{\frac{2}{k}}jY} \tilde{V}'\Phi_{j=-1+\frac{k}{2}(2-r_V)}\right)_{j-1/2,m} \,, \tag{4.17}$$

   where:

$$S'_{\pm 1/2} = e^{\mp\frac{i}{2}(H_1+H_2)+\frac{i}{2}\sqrt{3-\frac{2}{k}}Z - i\frac{1}{\sqrt{2k}}Y} \,, \tag{4.18}$$

   and $\tilde{V}'$ is a worldsheet anti-chiral operator which can be written as:

$$\begin{aligned} \tilde{V}' &= v e^{\frac{i}{a}(r_V-a^2)Z} \,, \\ \Delta'_V &= \Delta_V - r_V + \frac{a^2}{2} \,, \end{aligned} \tag{4.19}$$

   where again $a^2 = c_{\mathcal{N}/\,\mathrm{U}(1)}/3$. We thus see that $\tilde{\mathcal{Y}}_{1+k(r_V-2)}$ is nothing else than the complex conjugate of the operator $\mathcal{Y}_{1+k(r'_V-1)}$, where $r'_V = 3 - 2/k - r_V$ is the worldsheet R-charge of the complex conjugate of (4.19).

2. The above formulas for the weight and charge of $\tilde{V}'$ are exactly the ones given by spectral flow [18] in the $\mathcal{N}/\,\mathrm{U}(1)$ SCFT:

$$\begin{aligned} \Delta_\eta &= \Delta - r\eta + \frac{c}{6}\eta^2 \,, \\ r_\eta &= r - \frac{c}{3}\eta \,, \end{aligned} \tag{4.20}$$

   for $\eta = 1$. This transformation maps the NS sector of the theory to itself, and similarly for the Ramond sector.

3. The operators (4.14) correspond to the spacetime bottom components of the operators found in [10], which were shown to be top components of a chiral superfield. One can indeed act with the relevant spacetime supercharge on $\mathcal{Y}_{1+k(r_V-1)}$ and find an operator of the form (4.1), with the quantum numbers specified in subsection 4.1 of [10].

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To summarize, in the untwisted sector of the superstring on $AdS_3 \times \mathcal{N}$, we find three families of vertex operators corresponding to the bottom components of (anti-)chiral superfields in spacetime. For convenience, we rewrite them below:

$$\begin{aligned}
\mathcal{X}_{-k(1-r_V)} &= e^{-\varphi} e^{-i\sqrt{\frac{2}{k}}(j+1)Y} V \Phi_{j=\frac{k}{2}(1-r_V)-1,m} \qquad r_V + \frac{1}{k} < 1\,, \\
\mathcal{W}_{kr_V} &= e^{-\varphi} e^{i\sqrt{\frac{2}{k}}jY} V (\psi \Phi_{j=\frac{k}{2}r_V})_{j-1,m} \qquad r_V + \frac{1}{k} < 1\,, \\
\mathcal{Y}_{1+k(r_V-1)} &= e^{-\frac{\varphi}{2}} \left(S e^{i\sqrt{\frac{2}{k}}jY} V \Phi_{j=\frac{k}{2}(r_V-1)} \right)_{j-1/2,m} \qquad 1 < r_V + \frac{1}{k} < 2\,,
\end{aligned} \tag{4.21}$$

where V is a chiral operator of $\mathcal{N}/\,\mathrm{U}(1)$ with an R-charge $r_V \geq 0$. Their spacetime weights are in the following ranges:

$$\begin{aligned}
\frac{1}{2} &< h(\mathcal{X}) = \frac{k}{2}(1-r_V) \leq \frac{k}{2} \\
0 &\leq h(\mathcal{W}) = \frac{k}{2} r_V < \frac{k-1}{2} \\
0 &< h(\mathcal{Y}) = \frac{1}{2} + \frac{k}{2}(r_V - 1) < \frac{k}{2}\,.
\end{aligned} \tag{4.22}$$

4.2 The twisted sectors

We now turn to consider the chiral spectrum in the spacetime CFT which corresponds on the worldsheet to vertex operators obtained by twisting the operators (4.1), (4.6) and (4.10) by the twist fields $t^w_\pm$. The result will be that each one of the (anti-)chiral operators listed in (4.21) gives rise upon twisting to a family of (anti-)chiral operators whose spacetime weight is given by:

$$h_w = h_0 + \frac{k|w|}{2}\,, \tag{4.23}$$

where h_0 is the weight of the untwisted operators. The appearance of this pattern will prove important later for the identification of the spacetime theory.

In order to properly carry on the twist, it is useful to realize that there are essentially three pieces in the twist fields (3.14). The SL(2) part decomposes into a bosonic and a fermionic part, according to:

$$J^3 = -\sqrt{\frac{k}{2}}\partial X = j^3 - i\partial H_1 = -\sqrt{\frac{k+2}{2}}\partial x - i\partial H_1\,, \tag{4.24}$$

where all the scalars are canonically normalized. Therefore we can write the twist fields as:

$$t^w_\pm = e^{w\left(\sqrt{\frac{k+2}{2}}x + iH_1 \pm i\sqrt{\frac{k}{2}}Y\right)}\,. \tag{4.25}$$

Since the $\Phi_{jm\bar{m}}$ operators are purely bosonic, the expressions (2.7), (2.8) and (2.9) apply, but with k replaced by $k+2$, the bosonic level of SL(2) in the supersymmetric context. In the twisted operators however we take into account also the piece

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containing H_1, so that their total weight is:

$$\Delta(\Phi^w_{jm\bar{m}}) = \frac{-j(j+1)}{k} - \frac{k}{4}w^2 - mw\,. \tag{4.26}$$

On the other hand, the $\mathrm{U}(1)_Y$ part of the twist will generate the twisted operator $e^{i\left(q\pm w\sqrt{\frac{k}{2}}\right)Y}$ with weight:

$$\Delta_{\mathrm{U}(1)} = \frac{1}{2}q^2 \pm qw\sqrt{\frac{k}{2}} + \frac{k}{4}w^2\,. \tag{4.27}$$

We are now ready to twist a general operator. Starting from (4.1) and acting with $t^w_\pm$, we get:

$$\mathcal{X}^w_R = e^{-\varphi}e^{i\left(q\pm w\sqrt{\frac{k}{2}}\right)Y}V\Phi^w_{jm\bar{m}}\,. \tag{4.28}$$

The on-shell condition becomes:

$$-\frac{j(j+1)}{k} - mw + \frac{1}{2}q^2 \pm qw\sqrt{\frac{k}{2}} + \Delta_V = \frac{1}{2}\,, \tag{4.29}$$

while the GSO condition stays (4.3). The spacetime weight is:

$$h = -m - \frac{kw}{2}\,, \tag{4.30}$$

where again $w < 0$, and the R-charge is:

$$R = \sqrt{2k}q \pm kw\,. \tag{4.31}$$

Actually, had we straightforwardly twisted the physical operators (4.5) by t^w_+, the physicality condition (4.29) would boil down to imposing $m = -j-1$, giving rise to anti-chiral states in spacetime labelled by w.

The fact that physical twisted fields corresponding to (anti-)chiral operators in spacetime are the twist of (anti-)chiral physical fields is not surprising, since the twist operators $t^w_\pm$ have worldsheet weight zero and are themselves "(anti-)chiral," in the sense that their J^3 eigenvalue is the absolute value of half of their J^Y eigenvalue.

From the above discussion we thus see that starting with an operator of the list (4.21), taken to be in the $\mathcal{D}^-_j$ representation so that it creates an (anti-)chiral state in spacetime, the twist of a chiral state with t^w_- leads to a chiral state, and the twist of an anti-chiral state with t^w_+ leads to an anti-chiral state (remember that we take $w < 0$). The vertex operators obtained by twisting (4.5) by t^w_+ are:

$$\mathcal{X}^w_{-k(1-r_V)} = e^{-\varphi}e^{-i\sqrt{\frac{2}{k}}(j+1-\frac{kw}{2})Y}V\Phi^w_{j=\frac{k}{2}(1-r_V)-1,m=-j-1}\,, \qquad r_V + \frac{1}{k} - 1 < 0\,. \tag{4.32}$$

They correspond to anti-chiral operators in spacetime with weight:

$$h_w = \frac{k}{2}(1-r_V) + \frac{k|w|}{2} = -\frac{R}{2}\,. \tag{4.33}$$

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They clearly follow the pattern (4.23) with respect to the states created by (4.5). The vertex operators obtained by twisting (4.9) by t_-^w are:

$$\mathcal{W}^w_{kr_V} = e^{-\varphi} e^{i\sqrt{\frac{2}{k}}(j-\frac{kw}{2})Y} V(\psi\Phi^w_{j=\frac{k}{2}r_V})_{j-1,m=-j}\,, \qquad r_V+\frac{1}{k}-1<0\,. \tag{4.34}$$

Note that $(\psi\Phi_j)_{j-1,m=-j} = \psi^+\Phi_{j,m=-j-1}$ (ignoring pieces that annihilate the vacuum). They correspond to chiral operators in spacetime with weight:

$$h_w = \frac{k}{2}r_V + \frac{k|w|}{2} = \frac{R}{2}\,. \tag{4.35}$$

They also follow the pattern (4.23), with respect to the states created by (4.9). Twisting by t_-^w the vertex operators (4.14), one obtains:

$$\mathcal{Y}^w_{1+k(r_V-1)} = e^{-\frac{\varphi}{2}}\left(S e^{i\sqrt{\frac{2}{k}}(j-\frac{kw}{2})Y} V\Phi^w_{j=\frac{k}{2}(r_V-1)}\right)_{j-1/2,m=-j-1/2}\,, \qquad 1<r_V+\frac{1}{k}<2\,. \tag{4.36}$$

They correspond to chiral operators in spacetime with weight:

$$h_w = \frac{1}{2} + \frac{k}{2}(r_V-1) + \frac{k|w|}{2} = \frac{R}{2}\,. \tag{4.37}$$

They follow the pattern (4.23), with respect to the operators (4.14). More details on the derivation of (4.32), (4.34) and (4.36), and on the discussion which follows, are to be found in appendix A.

The vertex operators (4.32), (4.34) and (4.36) (together with their charge conjugates) give rise to the chiral spectrum of the spacetime theory obtained by the perturbative string analysis. Given the chiral spectrum of the worldsheet CFT on $\mathcal{N}/\,\mathrm{U}(1)$, each state being characterized by r_V, we get the vertex operators corresponding to (anti-)chiral states in spacetime listed in (4.21). Each one of the above operators is the $w=0$ member of a family of (anti-)chiral states forming a regular pattern of weights increasing by $k/2$ each time w is increased by 1. The vertex operators corresponding to these states are obtained by twisting the operators in (4.21) by the appropriate twist field.

In principle, one should also consider the possibility of twisting an (anti-)chiral vertex operator with the "wrong" twist field, since this is a possibility which is by no means excluded by the worldsheet CFT. To verify that these are not additional chiral operators we now show that vertex operators obtained in this way just lead to a reshuffling between, and among, the different families above.

For instance, twisting (4.5) by t_-^w, one can see that the physicality condition (4.29) will now impose $m=j+1$. We thus conclude that t_-^w has to act on $\mathcal{X}$ operators in the $\mathcal{D}_j^+$ representation. The outcome is:

$$\tilde{\mathcal{X}}^w_{-k(1-r_V)} = e^{-\varphi} e^{-i\sqrt{\frac{2}{k}}(j+1+\frac{kw}{2})Y} V\Phi^w_{j=\frac{k}{2}(1-r_V)-1,m=j+1}\,, \qquad r_V+\frac{1}{k}-1<0\,. \tag{4.38}$$

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The spacetime weight is now:

$$h_w = -j - 1 - \frac{kw}{2} = \frac{k}{2}(r_V - 1) + \frac{k|w|}{2} = \frac{R}{2} . \tag{4.39}$$

Note that the state is now chiral. The expression (4.39) is of course valid only for $w \leq -1$. Noting that $h_{-1} = k/2r_V$, we see that (4.39) actually follows the pattern (4.35) (with $|w|$ replaced by $|w| - 1$), that is, with respect to the operators (4.9).

This was already noted in the first remark after (2.31), that the $w = -1$ twist of an operator in the $\mathcal{D}_j^+$ representation is an operator in the $\mathcal{D}^-_{\tilde{j}=\frac{k}{2}-j-1}$ representation[9]. Namely in the case above we have that $\hat{\mathcal{D}}^{+,w=-1}_{j=\frac{k}{2}(1-r_V)-1} = \hat{\mathcal{D}}^-_{\tilde{j}=\frac{k}{2}r_V}$. We can thus straightforwardly write:

$$\begin{aligned} \tilde{\mathcal{X}}^{w=-1}_{-k(1-r_V)} &= e^{-\varphi} e^{i\sqrt{\frac{2}{k}}\tilde{j}Y} V e^{-iH_1} \Phi_{\tilde{j}=\frac{k}{2}r_V, m=-\tilde{j}-1} = \\ &= e^{-\varphi} e^{i\sqrt{\frac{2}{k}}\tilde{j}Y} V (\psi \Phi_{\tilde{j}=\frac{k}{2}r_V})_{\tilde{j}-1, m=-\tilde{j}} = \mathcal{W}_{kr_V} , \end{aligned} \tag{4.40}$$

where we used the fact that $e^{-iH_1} = \psi^+$.

Similarly, acting with $t_+^{w=-1}$ on $\mathcal{W}_{kr_V}$ one gets $\mathcal{X}_{-k(1-r_V)}$, and acting with $t_+^{w=-1}$ on $\mathcal{Y}_{1+k(r_V-1)}$ one gets $\tilde{\mathcal{Y}}_{1+k(r_V-2)} = (\mathcal{Y}_{1+k(r'_V-1)})^*$ (recall the discussion after (4.19)). We thus conclude that there are no new twisted vertex operators leading to (anti-)chiral states in spacetime, obtained by acting with the "wrong" twist field.

To summarize, twisting with $t_\pm^{w<0}$ the vertex operators in (4.21) we obtain the following results:

$$\begin{aligned} t_+^w(\mathcal{X}_{-k(1-r_V)}) &= \mathcal{X}^w_{-k(1-r_V)} , & t_-^w(\mathcal{X}_{-k(1-r_V)}) &= \mathcal{W}^{w+1}_{kr_V} , \\ t_-^w(\mathcal{W}_{kr_V}) &= \mathcal{W}^w_{kr_V} , & t_+^w(\mathcal{W}_{kr_V}) &= \mathcal{X}^{w+1}_{-k(1-r_V)} , \\ t_-^w(\mathcal{Y}_{1+k(r_V-1)}) &= \mathcal{Y}^w_{1+k(r_V-1)} , & t_+^w(\mathcal{Y}_{1+k(r_V-1)}) &= (\mathcal{Y}^{w+1}_{-1-k(r_V-2)})^* . \end{aligned} \tag{4.41}$$

Two related remarks are the following:

1. Assuming that the smallest spacetime CFT allowed by the formula (2.4) has $c = 6k$ ($\mathcal{M}_{6k}$ in (1.1)), we can now act on any chiral state in it with the $N = 2$, $\eta = 1$ spectral flow (4.20) and obtain $h \to c/6 - h = k - h$. The resulting operators clearly cannot come from the untwisted $w = 0$ sector – the short string – because of the bound (3.5) which forces $h \leq k/2$, see (4.22). One therefore needs to include the long string sector to recover the spectrum of the $c = 6k$ theory. Indeed we expect $\mathcal{M}_{6k}$ to be the spacetime CFT of a theory of a single long string, and the "missing" $N = 2$ chiral states (roughly half of them) could in principle "disappear" in the continuum (see the next remark).

2. In the discussion of the untwisted sector we did not consider the continuous representations $\mathcal{C}_{j,\mu}$ of SL(2), since they represent tachyons, and are eliminated

[9] Note that we have to replace k in the first comment after (2.31) with $k + 2$.

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by the GSO projection. However in the $w \neq 0$ sectors such representations are included in the physical spectrum; they correspond to the continuum associated with the long strings, as shown in [11]. For instance, the lowest lying physical operator surviving the GSO projection is:

$$e^{-\varphi}\Phi^{w=-1}_{j,m}\,, \qquad j=-\frac{1}{2}+is\,, \qquad m=\frac{k}{2}-\frac{(k-1)^2}{4k}-\frac{s^2}{k}\,. \tag{4.42}$$

(Recall that in this section Φ^w includes e^{iwH_1}, see near eq. (4.26).) In particular, the gap above which the continuum starts is $h=(k-1)^2/4k$ (as is obtained from (4.42) with $s=0$ using (4.30)), in agreement with [7]. It is not clear if and which of the operators in the continuum correspond to observables in the spacetime theory. It is possible in principle that some chiral operators could "disappear" in the continuum; this was argued for the particular example of $AdS_3 \times S^3 \times T^4$ in [7] (we return to this example later), and may also provide the missing chiral states discussed in the preceding remark. Here our study is limited to local observables in the discrete representations and their twists.

5. The candidate spacetime theory

The analysis in the previous sections, using perturbative worldsheet techniques, revealed that the chiral spectrum of the spacetime CFT obeys a simple and regular pattern of duplication (4.23). We will show that the same pattern appears in a class of orbifold CFTs – symmetric products – and we will thus argue that the $c=6kp$ spacetime CFT dual to superstring theory on a background of the general form $(AdS_3)_k \times \mathcal{N}$ is (a deformation of) a symmetric product CFT of the form:

$$(\mathcal{M}_{6k})^p/S_p\,. \tag{5.1}$$

In what follows we will give a brief review of the subject of symmetric product CFTs and discuss their chiral spectrum showing how the pattern (4.23) naturally appears in this setup.

In order to compare the chiral spectrum obtained from the worldsheet vertex operators – corresponding to single string states in spacetime – to states in (5.1), we need to distinguish between single and multi-string states from the spacetime perspective. In the two dimensional CFT (5.1), it is a priori subtle to identify single particle states due to the lack of asymptotic states and an S-matrix. Yet, as shown below, we argue following [15, 19, 20] that there is a natural identification of "single particle states" with operators in certain twisted sectors.

According to the general theory of orbifold CFTs, modding by a (discrete) non-Abelian symmetry group gives rise to twisted sectors labelled by the conjugacy classes of the group. In the case of a symmetric product $\mathcal{M}^p/S_p$ [21], the conjugacy classes

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of S_p are composed of disjoint cyclic permutations of various lengths. Specifically, each conjugacy class can be written as:

$$[C] = (N_1)^{n_1}(N_2)^{n_2}\cdots(N_r)^{n_r}\,, \tag{5.2}$$

with $\sum_{i=1}^{r} n_i N_i = p$ and where by (N) we denote a single disjoint Z_N cycle. Obviously conjugacy classes come in one-to-one correspondence with partitions of the integer p.

It is natural to associate "single particle states" with operators in a single Z_N twisted sector (more precisely, corresponding to conjugacy classes of the form $[C_N] = (N)(1)^{p-N}$). In particular, we also have the original single particle states of the 'diagonal' CFT $\mathcal{M}_{\rm diag} \cong \mathcal{M}$. Indeed, it turns out that the full (chiral) spectrum of a symmetric product CFT can be represented in a Fock space [19] (when $\mathcal{M}$ is a σ-model on a complex manifold). This Fock space is built of elementary oscillators associated with (chiral) states of the kind described above, i.e. chiral Z_N twist fields and chiral operators in the diagonal CFT, which are therefore identified as single particle states.

It thus turns out that the generators of the chiral spectrum of a symmetric product $\mathcal{M}^p/S_p$ (corresponding to "single particle states") are the chiral operators in the Z_N twisted sectors (i.e. the sectors of maximal length) of cyclic orbifolds $\mathcal{M}^N/Z_N$ with $N = 1, 2 \ldots p$ (symmetrized by S_p).[10]

5.1 The chiral spectrum of symmetric products

We now turn to computing the chiral spectrum of $\mathcal{M}^p/S_p$ theories corresponding to single particle states (a similar manipulation appeared in [20]).

First we recall facts about the $\mathcal{M}^N/Z_N$ orbifold (see appendix B for the details). Given an $N = 2$ SCFT $\mathcal{M}$, for any operator in $\mathcal{M}$ with weight h and R-charge R, there is an operator in the Z_N twisted sector of $\mathcal{M}^N/Z_N$ with weight and R-charge given by [22, 23]:

$$h^N = \frac{h}{N} + \frac{c}{24}\frac{N^2-1}{N}\,, \qquad R^N = R\,, \tag{5.3}$$

where c is the central charge of $\mathcal{M}$. Another property of $N = 2$ theories is the spectral flow [18] which relates any operator in the NS sector with weight h_{NS} and R-charge R_{NS} to an operator in the Ramond sector with (see (4.20) with $\eta = 1/2$):

$$\begin{aligned} h_R &= h_{NS} - \frac{1}{2}R_{NS} + \frac{c}{24}\,,\\ R_R &= R_{NS} - \frac{c}{6}\,. \end{aligned} \tag{5.4}$$

From the first line above one sees that chiral states of the NS sector correspond under spectral flow to Ramond ground states.

[10] The Z_1 chiral spectrum is that of $\mathcal{M}$ itself.

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Starting now with a chiral state in $\mathcal{M}$ with $h_{NS} = 1/2R_{NS} \equiv 1/2R$, it flows to the Ramond ground state specified by:

$$\begin{aligned} h_R &= \frac{c}{24}\,, \\ R_R &= R - \frac{c}{6}\,. \end{aligned} \tag{5.5}$$

Using (5.3) to compute the weight and the charge of the corresponding state in the Z_N twisted sector, we obtain:

$$\begin{aligned} h_R^N &= \frac{Nc}{24}\,, \\ R_R^N &= R - \frac{c}{6}\,. \end{aligned} \tag{5.6}$$

Note that in all generality the twist field corresponding to a Ramond ground state of $\mathcal{M}$ is also a Ramond ground state, but now of $\mathcal{M}^N/Z_N$. This assures us that flowing back to the NS sector we will end up with a chiral state of the orbifold CFT. Indeed, using (5.4) backwards and with Nc instead of c, we get:

$$\begin{aligned} h_{NS}^N &= \frac{1}{2}R + \frac{c}{12}(N-1)\,, \\ R_{NS}^N &= R + \frac{c}{6}(N-1)\,. \end{aligned} \tag{5.7}$$

We conclude that any chiral state in $\mathcal{M}$ gives rise to a chiral state in $\mathcal{M}^N/Z_N$.[11]

One can use the reasoning above backwards to show that any chiral state in $\mathcal{M}^N/Z_N$ corresponds in this manner to a chiral state in $\mathcal{M}$. This establishes a one-to-one correspondence between chiral states in the building block theory $\mathcal{M}$ (including the identity) and chiral states in each of the Z_N twisted sectors of $\mathcal{M}^p/S_p$.

Recall that the spectrum obtained in (5.7) corresponds to operators in $\mathcal{M}^p/S_p$ associated to single particle states. Comparing the spectrum (5.7) with $c = 6k$ to the worldsheet result (4.23), we see that the chiral spectra follow the same pattern. The operators in the $w = 0$ sector on the worldsheet define chiral operators of $\mathcal{M}_{6k}$ with $h \leq k/2 = c_{\mathcal{M}}/12$.[12,13] Identifying $|w| = N - 1$, the pattern in (5.7) coincides precisely with (4.23). For any operator in $\mathcal{M}$ with weight h which we find from the worldsheet, there is an operator in the $|w|$ twisted sector with weight $h + k/2(N-1)$ associated to the Z_N twisted sector of the symmetric product.

[11]Note that applying (5.3) directly to the original chiral state would generically lead to a twisted state that is not chiral.

[12]See the remarks at the end of section 4 on why we only see operators with $h \leq k/2$ and not $h \leq k$ as expected.

[13]The chiral spectrum of $\mathcal{M}_{6k}$ that we obtain from the worldsheet has a symmetry of reflection around $h = k/4$. The "generator" of this symmetry is actually the "wrong" twist with $w = -1$ (see the right column of (4.41)). It would be interesting to understand what kind of spacetime operator would generate that symmetry. Note that this cannot be a spectral flow (4.20) for any value of η (the symmetry generated by the $\eta = 1$ spectral flow is around $h = k/2$).

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6. $N = 4, 3, 2$ examples

We now illustrate the general structure presented in the previous sections in a few concrete examples, some of which were already studied in the literature.

6.1 Small $N = 4$

We start with the case of type II string theory on $AdS_3 \times S^3 \times T^4$. This background is described on the worldsheet by a product of WZW models $\mathrm{SL}(2)_k \times SU(2)_k \times U(1)^4$ and was shown to possess small $N = (4, 4)$ superconformal symmetry in spacetime [2]. As before, for simplicity we mainly discuss the left-moving sector.

In this case the spacetime $N = 2$ subalgebra is constructed [8] as reviewed in section 3.1 with respect to the $N = 2$ worldsheet CFT $\mathcal{N}/\,\mathrm{U}(1) = \mathrm{SU}(2)_k/\,\mathrm{U}(1) \times T^4$. Below we will first consider the $N = 2$ spacetime chiral primaries corresponding to the untwisted vertex operators (4.21); they actually turn out to be identical to the list of spacetime $N = 4$ chiral primaries as given in [4].

To construct the vertex operators (4.21), we first have to collect the chiral operators of $\mathrm{SU}(2)_k/\,\mathrm{U}(1) \times T^4$. They are generated by:

(i) The chiral operators of the $N = 2$ minimal model $\mathrm{SU}(2)_k/\,\mathrm{U}(1)$ with R-charge $r_i = i/k$ $(i = 0, \ldots, k-2)$.

(ii) The chiral operators of the SCFT on T^4: two complex fermions $\psi^\pm$ with R-charge $r_\pm = 1$, and the current J bilinear in the fermions, with $r = 2$.

The spacetime chiral operators constructed purely from the minimal model satisfy $r_i + 1/k < 1$, and thus give rise to operators $\mathcal{X}_{-k(1-r_i)} = \mathcal{X}_{-(k-i)}$ and $\mathcal{W}_{kr_i} = \mathcal{W}_i$ in eq. (4.21). Taking products of the minimal model operators with the $\psi^\pm$ chiral operators of T^4, we obtain operators which satisfy $1 < r_\pm + r_i + 1/k < 2$. They thus give rise to the vertex operators $\mathcal{Y}^\pm_{1+k(r_\pm + r_i - 1)} = \mathcal{Y}^\pm_{i+1}$. Note that the operators J do not give rise to any vertex operator in (4.21) since they do not fit in any one of the inequalities of (4.21).

To summarize, we get in the untwisted sector two towers of chiral states from the NS sector, one with spacetime weight $h = 0, 1/2, \ldots, (k-2)/2$ and the other with $h = 1, \ldots, (k-1)/2, k/2$, and two towers of states from the Ramond sector, both with weights $h = 1/2, 1, \ldots, (k-1)/2$. This is the same list of vertex operators given in [4]. Restoring $\mathrm{SU}(2)_k$ notations, these are given by (we parametrize the range of j by an integer $i = 0, \ldots, k-2$, in agreement both with the SL(2) and the $\mathrm{SU}(2)_k$ unitarity bounds):

$$\begin{aligned}
\mathcal{W}_j &= e^{-\varphi} V_j (\psi \Phi_j)_{j-1}\,, & h &= j = \frac{i}{2}\,,\\
\mathcal{Y}^\pm_j &= e^{-\frac{\varphi}{2}} (S^\pm V_j \Phi_j)_{j+1/2, j-1/2}\,, & h &= j + \frac{1}{2} = \frac{i+1}{2}\,,\\
\mathcal{X}_j &= e^{-\varphi} (\chi V_j)_{j+1} \Phi_j\,, & h &= j + 1 = \frac{i+2}{2}\,,
\end{aligned} \tag{6.1}$$

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where V_j is an operator of spin j in the $\mathrm{SU}(2)_k$ WZW and $(\chi V_j)_{j+1}$ is its combination with the $\mathrm{SU}(2)_k$ fermions which has spin $j+1$.

To include the w twisted sectors in a way that preserves not only the spacetime $N=2$ superconformal symmetry but also the $N=4$ one, we should keep those twisted vertex operators that are mutually local with all the generators of the $N=4$ superalgebra. Since in this case all the twist fields (3.14) are mutually local with the $N=2$ subalgebra and with the generators of the SU(2) R-symmetry, we must include all of the w twisted sectors. One then finds that the above towers continue and repeat themselves. Since the repetition pattern (4.23) has a step of $k/2$, we see that every tower has one state every k ones that is missing, in an otherwise linear series (we shall return to this point below).

String theory on $AdS_3 \times S^3 \times T^4$ can be obtained as the near horizon limit of k NS5-branes and p fundamental strings. By U-duality it is related to a system of a single NS5-brane and kp fundamental strings [7, 24], so that they belong to the same moduli space. Hence it is expected [25] that the spacetime theory should be a deformation of the σ-model on $(T^4)^{kp}/S_{kp}$ – the instanton moduli space of kp strings in a 5-brane.[14]

The chiral spectrum of such a $(T^4)^{kp}/S_{kp}$ orbifold is obtained as follows. The SCFT on T^4 has $N=4$ superconformal symmetry and its $N=2$ chiral operators listed above are highest weights of representations of the $N=4$ SU(2) R-symmetry. The $\psi^{\pm}$ are in the spin 1/2 and the J is in the spin 1 representation, their weight coinciding with their SU(2) spin. Applying the spectral flow procedure of section 5 to a chiral state of weight j_r we get in the Z_N twisted sector a chiral state with weight $h^N = j_r + (N-1)/2$. This gives rise to four towers of single particle chiral states, one with $h=(N-1)/2$, two with $h=N/2$ and one with $h=(N+1)/2$, where $N=1,2,\dots$ ($N=1$ is the diagonal T^4). Note that the above discussion is purely holomorphic. When combining left and right movers one should match only twist fields corresponding to the same N. Thus for a given N one has a total of 16 possibilities, 6 with $h=\bar{h}$, 8 with $h=\bar{h}\pm 1/2$ and 2 with $h=\bar{h}\pm 1$. These results can also be obtained by direct orbifold CFT analysis [27], or by using the Poincaré polynomial [15, 19, 20]. Since our general expectation is that the spacetime theory should be a (deformation of a) symmetric product $\mathcal{M}^p/S_p$, it is tempting to argue that $(T^4)^{kp}/S_{kp}$ lies in the same moduli space as $((T^4)^k/S_k)^p/S_p$. Indeed they have the same chiral spectrum corresponding to single particle states. Then we would conclude that $\mathcal{M}_{6k}$ is in the moduli space of $(T^4)^k/S_k$. This is not accidental and actually meshes nicely with the general picture. As explained above, string theory on $AdS_3 \times S^3 \times T^4$ is in the moduli space of the near horizon theory of a single NS5-brane and kp fundamental strings. As such, the spacetime theory is expected

[14] Actually, it is claimed [26, 33] that in this particular case there is an extra T^4 attached in a certain way to $(T^4)^{kp}/S_{kp}$. This extra piece is not “seen” by the present worldsheet considerations.

to be not only in the moduli space of $(\mathcal{M}_{6k})^p/S_p$, but also in the one of $(\mathcal{M}_6)^{kp}/S_{kp}$, leading to the conclusion that $\mathcal{M}_{6k}$ is in the moduli space of $(\mathcal{M}_6)^k/S_k$.

However, we now encounter an apparent difficulty since the spectrum computed from string theory in the untwisted sector (6.1), which we would associate to the spectrum of $\mathcal{M}_{6k}$, is actually not exactly the one of $(T^4)^k/S_k$ since we miss the states corresponding to the Z_k twisted sector ($i = k-1$ in (6.1)).

It was argued in [7] that these "missing" spacetime chiral states might be found within the continuum, namely, from the vertex operators constructed with continuous representations of SL(2) (i.e. from the long strings). The missing chiral states disappear in the continuum because the background we consider, with vanishing RR fields, is a singular locus of the moduli space, where the spacetime CFT is expected to be non-local.[15]

A possibly related difficulty of the worldsheet approach is that we cannot consider the case where we have a single NS5-brane, that is when $k = 1$, since $k \geq 2$ by unitarity of the SU(2) WZW model.

6.2 Large $N = 4$

We now turn to the case of type-II string theory on $AdS_3 \times S^3 \times S^3 \times S^1$ [3] which was shown to have the large $N = (4,4)$ superconformal algebra. This background is described on the worldsheet by a product of WZW models $\mathrm{SL}(2)_k \times \mathrm{SU}(2)_{k'} \times \mathrm{SU}(2)_{k''} \times \mathrm{U}(1)$. We will restrict to the case $k' = k'' = 2k$.

The untwisted chiral spectrum (4.21) in this case can be constructed similarly to the previous example, where now $\mathcal{N}/\,\mathrm{U}(1) = [(\mathrm{SU}(2) \times \mathrm{SU}(2))/\,\mathrm{U}(1)] \times \mathrm{U}(1)$ [8]. Unlike the small $N = 4$ case, in the present case only part of the $N = 2$ chiral operators are chiral primaries of the large $N = 4$ superalgebra. The latter consist of [3, 28]:

$$\begin{aligned}
\mathcal{W}_j &= e^{-\varphi} V_j' V_j'' (\psi \Phi_j)_{j-1}\,, \qquad h = j = \frac{i}{2}\,, \\
\mathcal{Y}_j &= e^{-\frac{\varphi}{2}} (S V_j' V_j'' \Phi_j)_{j+\frac{1}{2}, j+\frac{1}{2}, j-\frac{1}{2}}\,, \qquad h = j + \frac{1}{2} = \frac{i+1}{2}\,.
\end{aligned} \tag{6.2}$$

Operators of the form $\mathcal{X}_j$ in (6.1) do not belong to the large $N = 4$ chiral ring. Note that in the above the unitarity bound (3.5) on the SL(2) spin j implies a stricter bound $j < (k'-2)/4$ on the SU(2) spin than required by unitarity of the SU(2) WZW model.

Twisting the operators (6.2) with (3.14), we should keep only the resulting operators which are mutually local with the full large $N = 4$ superalgebra. Unlike the small $N = 4$ case, only the $w \in 2\mathbb{Z}$ twisted operators preserve the large supersym-

[15] Note that the missing $N = 4$ chiral primary in each tower is the first among a series of $N = 2$ chirals of $(T^4)^k/S_k$ with $k/2 \leq h \leq k$, which are expected to "disappear" in the continuum (see the remarks at the end of section 4).

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metry.[16] Therefore in the chiral spectrum we obtain the pattern:

$$h_w = h_0 + kn = h_0 + \frac{k'}{2}n\,, \qquad n = \frac{|w|}{2} \in \mathbb{Z}\,. \tag{6.3}$$

The $AdS_3 \times S^3 \times S^3 \times S^1$ background is obtained as the near horizon limit of two orthogonal stacks of k' NS5-branes and p fundamental strings along their intersection. As in the previous subsection, it can be shown that this background is in the moduli space of the background obtained from $k'p$ fundamental strings at the orthogonal intersection of two single NS5-branes [29]. Hence, similar to the small $N = 4$ case one expects that $\mathcal{M}_{6k}$ is in the moduli space of $(\mathcal{M}_3)^{k'}/S_{k'}$. In particular, $(\mathcal{M}_{6k})^p/S_p$ is in the moduli space of $(\mathcal{M}_3)^{k'p}/S_{k'p}$. Indeed, it was argued in [3] that the chiral spectrum (6.2) is compatible with the one of a symmetric product of a SCFT $\mathcal{M}_3$ which is a $c = 3$ theory of one scalar X and its four fermionic superpartners ψ^a, and which possesses large $N = 4$ superconformal symmetry.

The R-symmetry representation of operators in $(\mathcal{M}_3)^{k'p}/S_{k'p}$ is denoted by $\mathrm{SU}(2) \times \mathrm{SU}(2)$ spins (j', j''). Studying the $N = 4$ chiral ring we focus on $j' = j'' = j_r$ [30]. The chirality condition in this case is $h = 1/2(j' + j'') = j_r$. In the $\mathcal{M}_3$ model there is just one chiral operator coming from the $\mathrm{SO}(4)$ vector of the fermions ψ^a. Starting with a chiral state in $\mathcal{M}_3$ with $h = j_r$ and applying the $(N = 2)$ spectral flow transformation of section 5 we get in the Z_N twisted sector a chiral state with $h^N = j_r + (N-1)/4$. Clearly, only when N is odd $h^N \in 1/2\mathbb{Z}$ and the twisted operators are in the chiral ring (when N is even, $h^N \in 1/2\mathbb{Z} + 1/4$ which corresponds to $j'_N = j''_N \pm 1/2$, see [3]). Thus we find two towers of chiral single particle states, one with $h = (N-1)/4 = i/2$ and the other with $h = (N+1)/4 = (i+1)/2$, for $N = 2i + 1$ with $i = 0, 1, 2, \ldots$

Finally, we should mention that as in the previous case the last chiral operators of $\mathcal{M}_{6k}$ (and hence of each Z_N twisted sector of $(\mathcal{M}_{6k})^p/S_p$) are missing in the worldsheet picture, with respect to the $(\mathcal{M}_3)^{k'}/S_{k'}$ candidate. Presumably they "disappear" in the continuum as in the small $N = 4$ case [7].

6.3 $N = 3$

In [12] superstring theory on a background of the form $AdS_3 \times G/H$ was considered, and shown to possess $N = 3$ superconformal symmetry in the dual spacetime CFT in two cases: $AdS_3 \times \mathrm{SU}(3)/\,\mathrm{U}(1)$ and $AdS_3 \times \mathrm{SO}(5)/\,\mathrm{SO}(3)$. In [31] the $N = 3$ chiral spectrum of the dual CFTs is analyzed using perturbative worldsheet methods, in the untwisted sector.

The physical vertex operators found in [31] are:

$$e^{-\phi} U_{[r,s],j'=\frac{r+s}{2}} (\psi \Phi_{j=\frac{r+s}{4}})_{j-1,m}\,, \qquad h = j = \frac{r+s}{4}\,, \tag{6.4}$$

where the G/H part of the vertex operator is labeled by the representation $[r, s]$ of

[16] Note that there will still be operators in the w odd sectors, but not chiral ones.

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$G = \mathrm{SU}(3), \mathrm{SO}(5)$ and the representation j' of the unbroken SU(2), which coincides with the spacetime R-symmetry. We also have that $j < (k-1)/2$ so that $j' < (k'/4) - 1$, where $k' = 4k$ is the level of SU(3) or SO(5) (and of the unbroken SU(2)). The above operators are chiral in spacetime since $h = j'/2$. For a given R-spin j', we find $2j'+1$ distinct chiral operators in the NS sector. There are no physical vertex operators coming from the Ramond sector giving additional spacetime $N=3$ chiral operators.

Again the physical vertex operators (6.4) fall into the classification of (4.21) with $\mathcal{N}/\mathrm{U}(1) = \mathrm{SU}(3)/\mathrm{U}(1)^2$ or $\mathcal{N}/\mathrm{U}(1) = \mathrm{SO}(5)/[\mathrm{U}(1)\times \mathrm{SO}(3)]$ where (6.4) is of the type $\mathcal{W}$, and the vertex operators of the form $\mathcal{X}, \mathcal{Y}$ give rise to chirals of $N = 2$ that are not chirals of $N = 3$. Including the twisted sectors, only the even w twisted operators are $N = 3$ chiral.

Unlike the $N = 4$ examples, $\mathcal{M}_{6k=3/2k'}$ cannot be itself in the moduli space of some symmetric product of a smaller building block, even though arithmetically it could be of the form $(\mathcal{M}_{3/2})^{k'}/S_{k'}$. The reason is that the chiral spectrum has for a specific R-charge j' a multiplicity of $2j'+1$, while in a symmetric product the multiplicity at every level is bounded from above by the total number of chiral operators in the building block theory, as it can be deduced from (5.7).

Note also that the arguments of [7] in favor of a spacetime CFT based on a more general $S_{k'p}$ orbifold rely on U-duality and the simple brane picture of the $N = 4$ cases and possibly do not apply here.

6.4 $N = 2$

As a last example, we consider the family of "non-critical" superstrings on $AdS_3 \times \mathcal{N}$, where $\mathcal{N}/\mathrm{U}(1) = \mathrm{SU}(2)_n/\mathrm{U}(1)$ is an $N = 2$ minimal model at level n (see [10, section 4.1]). The relation between n and the level of the SL(2) WZW model is:

$$k = \frac{n}{n+1}\,. \tag{6.5}$$

The chiral operators of $\mathcal{N}/\mathrm{U}(1)$ have R-charge $r_i = i/n$, with $i = 0, 1, \ldots, n-2$. They satisfy $1 < r_i + 1/k < 2$, so that they give rise to vertex operators of the kind $\mathcal{Y}_{1+k(r_i-1)} = \mathcal{Y}_{(i+1)(n+1)}$. Note that the identity in spacetime is given by an operator of the kind $\mathcal{W}_0$ where the $j = 0$ representation is the SL(2) identity. The vertex operators coming from the twisted sectors are straightforwardly obtained.

In this case the spacetime theory is a deformation of $(\mathcal{M}_{6n/(n+1)})^p/S_p$. The CFT $\mathcal{M}_{6n/(n+1)}$ cannot be in the moduli space of a symmetric product, even arithmetically, because its central charge is not linearly "quantized."

7. Discussion

In this work we studied a large family of $N = 2$ superstrings on $(AdS_3)_k \times \mathcal{N}$, with $g_s^2 \sim 1/p$ (1.2). In particular, we argued that the spacetime theory is in the moduli space of $(\mathcal{M}_{6k})^p/S_p$ (1.1), where $\mathcal{M}_{6k}$ is the $c = 6k$, $N = 2$ CFT whose Hilbert

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space corresponds to the worldsheet vertex operators of short strings, and a single long string. The short string vertex operators give rise to local observables in $\mathcal{M}_{6k}$. On the other hand, the long string contribution to the spectrum of $\mathcal{M}_{6k}$ is expected to consist of non-local observables – the continuum. This is compatible with the expectation [7] that string theory on AdS_3 with a vanishing RR background gives rise to a (singular) non-local CFT in spacetime.

Our study used standard perturbative worldsheet techniques. Needless to say that those are reliable only in the weak g_s limit, namely, when $p \gg k$. Strong string coupling effects become important for operators whose spacetime weights h approach the order of p. Yet, although we cannot trust the perturbative worldsheet results concerning such operators, the AdS_3/CFT_2 correspondence allows us to speculate about some non-perturbative properties.

For instance, the fact that the spacetime central charge is $c_{st} = 6kp$ implies that the chiral spectrum must be cut at $h = kp$. This means that the spectrum corresponding to single string states is cut at the $|w| = p-1$ twisted sector. Adding n string states with $n \leq p$ and with the maximal value of the total w equal to p will give rise to the chiral spectrum of a symmetric product with precisely p copies of $\mathcal{M}_{6k}$, as allowed by $c_{st} = 6kp$.

Moreover, "extrapolating" to small p, the general picture described in this work still seems to make sense, even though at present we cannot support it with direct computations.

For instance, for $p = 1$ we claim that the spacetime theory (1.1) is $\mathcal{M}_{6k}$. We expect that the $AdS_3 \times \mathcal{N}$ string background in the $p = 1$ case is constructed in the near horizon of a single fundamental string, in which case it can emit at most a single long string. Indeed, the spacetime theory $\mathcal{M}_{6k}$ is defined to correspond to a single long string and its short string excitations. More precisely, the short string bulk excitations consist of the $w = 0$ sector, while the continuum in the $|w| = 1$ sector (see the remarks at the end of section 4) corresponds to the long string [7, 11]. The $w = 0$ discrete states may be regarded as long string excitations emitted into the bulk [7].

For $p > 1$, the operators in the Z_N twisted sectors of $(\mathcal{M}_{6k})^p/S_p$ are associated with N long strings, similar to "matrix strings" [32]. Indeed, from the worldsheet perspective, the $|w| = N-1$ twists of operators in $\mathcal{M}_{6k}$ are associated with N long strings. The largest cyclic twisted sector Z_p corresponds to the maximal number of long strings p – the number of fundamental strings used to generate the AdS_3 background.

In sections 6.1 and 6.2, in the $N = 4$ examples, we saw that $\mathcal{M}_{6k}$ itself is in the moduli space of a symmetric product, $(\mathcal{M}_6)^k/S_k$ and $(\mathcal{M}_3)^{k'}/S_{k'}$, respectively. This is not accidental and meshes nicely with the general picture. In these cases the moduli space includes a point corresponding to the background constructed out of $P = kp$ and $k'p$ fundamental strings, respectively, thus leading to a maximal number

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of P long strings and a spacetime CFT in the moduli space of a symmetric product of P elementary blocks. This property is related to the rich U-duality and the simple brane picture associated with such high supersymmetry examples. We also expect $\mathcal{M}_{6k}$ to be in the moduli space of a symmetric product in backgrounds obtained by orbifolding of the $N = 4$ examples, for instance, $AdS_3 \times (S^3 \times S^3 \times S^1)/Z_2$ [5]. It would be interesting to understand which generic backgrounds $\mathcal{N}$ lead to $\mathcal{M}_{6k}$ which are themselves in the moduli space of a symmetric product.

Finally, we should mention that the pattern we find is not restricted only to the chiral spectrum. Repeating the computation of section 5.1 for an operator in $\mathcal{M}_{c=6k}$ with weight h and R-charge R such that:

$$h - \frac{1}{2}R = \delta\,, \tag{7.1}$$

one finds in the Z_N twisted sector of $\mathcal{M}^p/S_p$ an operator with:

$$\begin{aligned} h^N - \frac{1}{2}R^N &= \frac{\delta}{N}\,, \\ R^N &= R + \frac{c}{6}(N-1)\,. \end{aligned} \tag{7.2}$$

The worldsheet theory leads to the same pattern: twisting an operator in the $w = 0$ sector which corresponds in spacetime to an operator with h and R satisfying (7.1), we get an operator with:

$$\begin{aligned} h^w - \frac{1}{2}R^w &= f(\delta)\,, \\ R^w &= R + k|w|\,, \end{aligned} \tag{7.3}$$

where $f(\delta)$ depends on the particular operator we twist, and is such that $f(0) = 0$. When $\delta > 0$, generically $f(\delta) \neq \delta/N$ because the 2-d CFT dual to the superstring on $AdS_3 \times \mathcal{N}$, although it is conjectured to be in the moduli space of $\mathcal{M}^p/S_p$, is not at the symmetric product point.

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A. Detailed computation of some physical vertex operators

We give here in detail some of the computations underlying the results of sections 3 and 4. The main object is finding physical vertex operators, both in the NS and Ramond sectors, and both in the "untwisted" ($w = 0$) and "twisted" sectors.

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A.1 The BRST invariant supercharges

We briefly repeat here the derivation [8] of the physical $N = 2$ spacetime supercharges, using the notations of the present paper. In the process we introduce the worldsheet supercurrent which we will also use later to impose BRST invariance of the Ramond sector vertex operators.

The worldsheet $N = 1$ supercurrent is given by:

$$G_{\rm tot} = G_{\rm SL(2)} + G_{\rm U(1)} + G_{\mathcal{N}/\,\rm U(1)}\,, \tag{A.1}$$

where, using the notation of section 3 and recalling that $\epsilon^{123} = 1$:

$$\begin{aligned} G_{\rm SL(2)} &= \sqrt{\frac{2}{k}}\left(\psi^a j_a - \frac{i}{6}\epsilon_{abc}\psi^a\psi^b\psi^c\right) = \sqrt{\frac{2}{k}}\left(\psi^a j_a + i\psi^1\psi^2\psi^3\right)\,, \\ G_{\rm U(1)} &= \chi^Y J^Y = i\chi^Y \partial Y\,, \end{aligned} \tag{A.2}$$

and we can decompose $G_{\mathcal{N}/\,\rm U(1)}$ according to the $N = 2$ worldsheet superconformal symmetry of $\mathcal{N}/\,\rm U(1)$, i.e.:

$$G_{\mathcal{N}/\,\rm U(1)} = G^+ + G^- = g^+ e^{i\frac{Z}{a}} + g^- e^{-i\frac{Z}{a}}\,, \tag{A.3}$$

where Z is defined in (3.8), $g^{\pm}$ have regular OPE with Z, and:

$$a^2 = 3 - \frac{2}{k}. \tag{A.4}$$

To construct physical supercharges like (3.10) we need to pick spin fields S such that $e^{-\varphi/2}S$ is BRST invariant, which translates to requiring that there is no $z^{-3/2}$ singular term in the OPE of G_{tot} with S. Using the bosonization (3.9), a general spin field S is given by:

$$S_r^{\epsilon_2\epsilon_Z} \propto e^{-ir(H_1+\epsilon_2 H_2)+i\epsilon_Z(\frac{a}{2}Z-\frac{1}{\sqrt{2k}}Y)}\,, \tag{A.5}$$

where $r = \pm 1/2$, $\epsilon_2, \epsilon_Z = \pm 1$. The exact (relative) prefactors and cocycle factors are determined as follows. The total SL(2) currents after bosonization are:

$$\begin{aligned} J^3 &= j^3 - i\partial H_1\,, \\ J^{\pm} &= j^{\pm} \pm c_1 c_2 e^{\mp iH_1}\left(e^{iH_2} - e^{-iH_2}\right)\,, \end{aligned} \tag{A.6}$$

where c_1, c_2 are the cocycles associated to H_1 and H_2. For the spin fields $S_r^{\epsilon_2\epsilon_Z}$ to have the OPEs with the SL(2) currents appropriate to $j = 1/2$ and $m = r = \pm 1/2$ (see for instance appendix A of [4]), we take the prefactors to be such that $S_r^{\epsilon_2\epsilon_Z}$ take the form:

$$\begin{aligned} S_{1/2}^{\epsilon_2\epsilon_Z} &= e^{-\frac{i}{2}(H_1+\epsilon_2 H_2)+i\epsilon_Z(\frac{a}{2}Z-\frac{1}{\sqrt{2k}}Y)}\,, \\ S_{-1/2}^{\epsilon_2\epsilon_Z} &= -\epsilon_2 c_1 c_2 e^{\frac{i}{2}(H_1+\epsilon_2 H_2)+i\epsilon_Z(\frac{a}{2}Z-\frac{1}{\sqrt{2k}}Y)}\,. \end{aligned} \tag{A.7}$$

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Since from (A.3) one sees that $G_{\mathcal{N}/\,\mathrm{U}(1)}$ has at most a $z^{-1/2}$ singularity with the spin field (A.5), we focus on the $\mathrm{SL}(2)\times \mathrm{U}(1)$ part of the worldsheet supercurrent, which we can rewrite in a bosonized form:

$$\begin{aligned} G_{\mathrm{SL}(2)} + G_{\mathrm{U}(1)} = & \frac{1}{\sqrt{k}}\left[c_1 e^{iH_1} j^+ + c_1 e^{-iH_1} j^- - c_2\left(e^{iH_2} - e^{-iH_2}\right) j^3\right] + \\ & + \frac{1}{\sqrt{k}}\left[ic_2\left(e^{iH_2} - e^{-iH_2}\right)\partial H_1 + ic_2\left(e^{iH_2} + e^{-iH_2}\right)\sqrt{\frac{k}{2}}\partial Y\right]. \end{aligned} \tag{A.8}$$

For finding BRST invariant supercharges, only the second line of (A.8) gives potentially singular terms, and we can neglect all the cocycles. One then finds:

$$G_{tot}(z) S_r^{\epsilon_2 \epsilon_Z}(z') \sim \frac{1}{2\sqrt{k}}(z-z')^{-3/2}(-\epsilon_2 - \epsilon_Z) S_r^{-\epsilon_2, \epsilon_Z}(z') + \cdots . \tag{A.9}$$

We thus conclude that the physical supercharges are built from spin fields such that $\epsilon_2 \epsilon_Z = -1$, which is the result (3.11). Note that in this way the BRST condition also fixes the GSO projection on the supercharges, which amounts to fixing the sign of $\epsilon_2 \epsilon_Z$.

A.2 Untwisted sector physical vertex operators

We now look for physical vertex operators leading to (anti-)chiral operators in spacetime. We start from the NS sector, and show that vertex operators like (4.1):

$$\mathcal{X} = e^{-\varphi} e^{iqY} V \Phi_{jm}\,, \tag{A.10}$$

only lead to anti-chiral operators in spacetime. Here we take V to be an operator in $\mathcal{N}/\,\mathrm{U}(1)$ with R-charge $r_V \geq 0$, so that it can be rewritten as $V = v e^{i r_V / a Z}$. In order for (A.10) to survive the GSO projection, it must be mutually local with the supercharges (3.10)–(3.11). This imposes $-1/2 \pm (r_V/2) \mp (q/\sqrt{2k}) \in \mathbb{Z}$, which is (4.3). Physicality is then obtained imposing the on-shell condition (4.2). Since we are interested in (anti-)chiral operators in spacetime, we further impose that the spacetime weight of (A.10) equals half of the absolute value of its spacetime R-charge, leading to (4.4). Moreover, quantities appearing in (4.2), (4.3) and (4.4) must satisfy two inequalities, the bound on j (3.5) and $\Delta_V \geq r_V/2$. Combining now (4.4) and (4.2), we obtain j in terms of Δ_V:

$$j + 1 = \frac{k}{2}(1 - 2\Delta_V)\,. \tag{A.11}$$

The bound $j > -1/2$ imposes that $\Delta_V < 1/2\,(1 - 1/k) < 1/2$. Requiring now the GSO condition (4.3), we get:

$$\pm(1 - 2\Delta_V) - r_V = 2n + 1\,, \tag{A.12}$$

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where n is an integer. Choosing the $+$ sign, so that the spacetime operator would be chiral, we get $\Delta_V = n' - r_V/2$, with $n' \geq 1$. However $\Delta_V \geq r_V/2$ implies $r_V \leq n'$ so that $\Delta_V \geq n' - n'/2 = n'/2 \geq 1/2$, in contradiction with $\Delta_V < 1/2$. Therefore starting with an operator V with positive R-charge it is impossible to get a chiral spacetime operator.

Choosing thus the $-$ sign in (A.12), one gets $\Delta_V = n'' + r_V/2$, with $n'' \geq 0$. Clearly $\Delta_V < 1/2$ imposes $n'' = 0$ and thus V must be a worldsheet chiral operator. Moreover, its R-charge has to satisfy the bound $r_V < 1 - 1/k$, which is listed in (4.5).

The conclusion is that physical vertex operators like (A.10) only lead to anti-chiral operators in spacetime, with spacetime R-charge given by $\sqrt{2k}q = -2(j+1) = -k(1-r_V)$. To obtain spacetime chiral operators from vertex operators of the form (A.10), we have to take an anti-chiral operator $\tilde{V}$ of the $\mathcal{N}/\,\mathrm{U}(1)$ worldsheet CFT. It amounts to taking the complex conjugate of (4.5).

For vertex operators like (4.6), where:

$$(\psi\Phi_j)_{j-1,m} = \psi^+\Phi_{j,m-1} + \psi^-\Phi_{j,m+1} - 2\psi^3\Phi_{j,m}\,, \tag{A.13}$$

the way one obtains the final result (4.9) is very similar to the one above. The presence of the SL(2) fermions only modifies the GSO and the on-shell conditions to (4.8) and (4.7), and the spacetime weight. Otherwise the reasoning is identical, giving (for $r_V \geq 0$) only spacetime chiral operators.

In the Ramond sector, we now show that the BRST condition is non-trivial and actually fixes the spacetime chiral nature of (4.14). We start from the vertex operators:

$$\mathcal{Y} = e^{-\frac{\varphi}{2}}(Se^{iqY}V\Phi_j)_{j-1/2,m}\,, \tag{A.14}$$

where the presence of a general spin field of the form (A.5) affects the total Y and Z charges, and we have taken the SL(2) combination:

$$(S\Phi_j)_{j-1/2,m} = S_{1/2}\Phi_{j,m-1/2} - S_{-1/2}\Phi_{j,m+1/2}\,. \tag{A.15}$$

Rewriting the spin field $S_r^{\epsilon_2\epsilon_Z} = s_r^{\epsilon_2} e^{i\epsilon_Z(\frac{a}{2}Z - \frac{1}{\sqrt{2k}}Y)}$, we can expand (A.14) as:

$$\mathcal{Y} = e^{-\frac{\varphi}{2}} e^{i(q-\frac{\epsilon_Z}{\sqrt{2k}})Y} v e^{i(\frac{r_V}{a}+\epsilon_Z\frac{a}{2})Z}(s\Phi_j)_{j-1/2,m}\,. \tag{A.16}$$

Its on-shell condition is:

$$\frac{3}{8} + \frac{1}{2}\left(q - \frac{\epsilon_Z}{\sqrt{2k}}\right)^2 + \Delta_V - \frac{r_V^2}{2a^2} + \frac{1}{2}\left(\frac{r_V}{a} + \epsilon_Z\frac{a}{2}\right)^2 + \frac{1}{4} - \frac{j(j+1)}{k} = 1\,. \tag{A.17}$$

It can be rearranged as:

$$-\frac{1}{k}\left(j + \frac{1}{2}\right)^2 + \frac{1}{2}\left(q - \frac{\epsilon_Z}{\sqrt{2k}}\right)^2 + \Delta_V + \epsilon_Z\frac{r_V}{2} = 0\,. \tag{A.18}$$

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Requiring that (A.14) be (anti-)chiral in spacetime, we get the condition:

$$j+\frac{1}{2}=\pm\sqrt{\frac{k}{2}}\left(q-\frac{\epsilon_Z}{\sqrt{2k}}\right), \tag{A.19}$$

so that the on-shell condition (A.18) boils down to $\Delta_V=-\epsilon_Z r_V/2$. Since we chose $r_V\geq 0$, we see that we must take $\epsilon_Z=-1$ and require that V be a chiral operator of $\mathcal{N}/\,\mathrm{U}(1)$.

We will now impose the BRST condition on (A.14), taking into account that V is chiral. This entails that $G_{\mathcal{N}/\,\mathrm{U}(1)}(z)V(z')\sim G^-(z)V(z')\sim(z-z')^{-1}V'(z')$. On the other hand, since $\epsilon_Z=-1$, we have that $G^-(z)e^{-i\frac{a}{2}Z}(z')\sim(z-z')^{1/2}W(z')$, so that again $G_{\mathcal{N}/\,\mathrm{U}(1)}$ does not give terms relevant to the BRST condition when acting on (A.14).

We can rewrite once more (A.14), taking now into account (A.15) and (A.7):

$$\mathcal{Y}=e^{-\frac{\varphi}{2}}e^{i(q+\frac{1}{\sqrt{2k}})Y}e^{-i\frac{a}{2}Z}V\left[e^{-\frac{i}{2}(H_1+\epsilon_2H_2)}\Phi_{j,m-1/2}+\epsilon_2c_1c_2e^{\frac{i}{2}(H_1+\epsilon_2H_2)}\Phi_{j,m+1/2}\right], \tag{A.20}$$

and take the OPE with $G_{rest}=G_{\mathrm{SL}(2)}+G_{\mathrm{U}(1)}$ as given in (A.8). The result is:

$$G_{rest}(z)\mathcal{Y}(z')\sim\frac{1}{\sqrt{k}}(z-z')^{-3/2}\left[\epsilon_2(j+1)+\frac{1-\epsilon_2}{2}+\sqrt{\frac{k}{2}}q\right]\mathcal{Y}'(z'), \tag{A.21}$$

where:

$$\mathcal{Y}'=e^{-\frac{\varphi}{2}}e^{i(q+\frac{1}{\sqrt{2k}})Y}e^{-i\frac{a}{2}Z}V\left[c_2e^{-\frac{i}{2}(H_1-\epsilon_2H_2)}\Phi_{j,m-1/2}-\epsilon_2c_1e^{\frac{i}{2}(H_1-\epsilon_2H_2)}\Phi_{j,m+1/2}\right]. \tag{A.22}$$

Thus for $\epsilon_2=-1$ the BRST condition becomes $j=\sqrt{k/2}q$, that is the vertex operator (A.14) is spacetime chiral, while for $\epsilon_2=1$ the BRST condition is $j+1=-\sqrt{k/2}q$, so that the vertex operator (A.14) is spacetime anti-chiral.

Taking $\epsilon_2=-1$, which is the choice in (4.10), (4.11), we now show that the GSO projection fixes the spacetime R-charge. First of all mutual locality of (4.10), (4.11) with the supercharges (3.10), (3.11) is:

$$-\frac{1}{4}+rs\mp rs\pm\frac{a^2}{4}\mp\frac{r_V}{2}\pm\frac{1}{2k}\pm\frac{q}{\sqrt{2k}}\in\mathbb{Z}, \tag{A.23}$$

which translates to $\sqrt{2/k}q-r_V\in 2\mathbb{Z}+1$, as in (4.13). This leads to:

$$\frac{2j}{k}-r_V=2n+1\,,\qquad n\in\mathbb{Z}\,. \tag{A.24}$$

Taking into account the bound on j (3.5) and that $0\leq r_V\leq 3-2/k=(c_{\mathcal{N}/\,\mathrm{U}(1)})/3$, we get that n has to satisfy the bound $-2+1/2k<n<-1/2k$ which implies $n=-1$. We thus finally obtain $j=k/2(r_V-1)$, and the physical operators (4.14) have spacetime R-charge $\sqrt{2k}q+1=2j+1=1+k(r_V-1)$ (provided r_V satisfies the bound (4.15)).

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Focusing now on operators which are spacetime anti-chirals, we take $\epsilon_2 = 1$ in (A.20). The GSO condition changes slightly, i.e. the third term of (A.23) has the opposite sign. This leads to the condition $\sqrt{2/k}q - r_V \in 2\mathbb{Z}$. Repeating a very similar reasoning as above, we obtain the operators (4.16) with $j+1 = k/2(2-r_V)$ and a spacetime R-charge of $\sqrt{2k}q + 1 = 1 + k(r_V - 2)$. It is shown in section 4 that these operators do not define a new family of operators, but are actually complex conjugates of already existing operators of the kind (4.14).

A.3 Twisted sector physical vertex operators

In section 4.2 we learn that spacetime (anti-)chiral operators coming from the world-sheet twisted sectors are obtained twisting physical vertex operators in the untwisted sector, which lead to (anti-)chiral operators in spacetime. Here we show in more detail how the on-shell condition of the twisted vertex operators is enforced.

Starting with the operators (4.5) and twisting them with $t^w_\pm$, we obtain operators:

$$e^{-\varphi} e^{i\sqrt{2/k}(-j-1\pm\frac{k}{2}w)Y} V \Phi^w_{jm}\,, \tag{A.25}$$

which are on-shell provided:

$$\frac{1}{k}\left(-j-1\pm\frac{k}{2}w\right)^2 + \frac{r_V}{2} - \frac{j(j+1)}{k} - mw - \frac{kw^2}{4} = \frac{1}{2}\,, \tag{A.26}$$

where we have used (4.26). Note that the GSO condition is unchanged since the twist fields are mutually local with the supercharges by construction. Using the relation between j and r_V in (4.5), (A.26) boils down to:

$$m = \mp(j+1)\,. \tag{A.27}$$

Note that the spacetime weight is $h = -m - k/2w = \pm(j+1) - k/2w$ (recall that $w < 0$ for ingoing states) and the R-charge is $R/2 = -(j+1) \pm k/2w = \mp h$, so that twisting an operator (4.5) (which is anti-chiral in spacetime) with t^w_+ leads to an anti-chiral operator, while twisting it with t^w_- leads to a chiral operator in spacetime. Moreover (A.27) also tells us that the original Φ_{jm} must be built respectively from a $\mathcal{D}^-_j$ and a $\mathcal{D}^+_j$ representation of SL(2). In section 4.2 it is shown that the operators above twisted by t^w_- can be reconducted to the twist of operators of the kind (4.9).

Twisting the operators (4.9) we obtain:

$$e^{-\varphi} e^{i\sqrt{2/k}(j\pm\frac{k}{2}w)Y} V (\psi\Phi^w_j)_{j-1,m}\,, \tag{A.28}$$

where $(\psi\Phi^w_j)_{j-1,m}$ is given by (A.13) with Φ_{jm} replaced by Φ^w_{jm}. The GSO projection is again unchanged, however the on-shell condition is slightly complicated by the fact that $\Phi^w_{jm} \propto e^{iwH_1} e^{\sqrt{2/(k+2)}(m+\frac{(k+2)}{2}w)x}$ (see (4.25)) and thus depends on H_1. Using also $\psi^\pm = e^{\mp iH_1}$, we obtain that:

$$\Delta[(\psi\Phi^w_j)_{j-1,m}] = \frac{1}{2} - \frac{j(j+1)}{k} - mw - \frac{kw^2}{4}\,. \tag{A.29}$$

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Then the on-shell condition is:

$$\frac{1}{k}\left(j \pm \frac{k}{2}w\right)^2 + \frac{r_V}{2} - \frac{j(j+1)}{k} - mw - \frac{kw^2}{4} = 0\,, \tag{A.30}$$

and leads to $m = \pm j$. This value is allowed since $(\psi\Phi^w_j)_{j-1,m=\pm j} = \psi^{\mp}\Phi^w_{j,\pm(j+1)}$, while the other terms in (A.13) annihilate the vacuum. In this case too twisting with t^w_+ or t^w_- leads respectively to anti-chiral and chiral operators in spacetime. The identifications between twisted operators of the different kinds is listed in (4.41). Twisting now the Ramond sector operators (4.14), we obtain:

$$e^{-\frac{\varphi}{2}} e^{i\sqrt{\frac{2}{k}}(j+1/2\pm\frac{k}{2}w)Y} e^{-i\frac{a}{2}Z} V(s\Phi^w_j)_{j-1/2,m}\,, \tag{A.31}$$

where:

$$\begin{aligned}(s\Phi^w_j)_{j-1/2,m} = {} & e^{-i(1/2-w)H_1+\frac{i}{2}H_2}\Psi_{j,m-1/2}e^{i\sqrt{\frac{2}{k+2}}(m-1/2+\frac{k}{2}w)x} - \\ & - c_1c_2 e^{i(1/2+w)H_1-\frac{i}{2}H_2}\Psi_{j,m+1/2}e^{i\sqrt{\frac{2}{k+2}}(m+1/2+\frac{k}{2}w)x}\,.\end{aligned} \tag{A.32}$$

The on-shell condition leads to:

$$m = \pm\left(j + \frac{1}{2}\right)\,. \tag{A.33}$$

The resulting operators are still BRST invariant, as one can check with (A.8), using (2.27) and discarding all operators which annihilate the vacuum. Again the twisted operators are identified in (4.41), and discussed in section 4.2.

We have thus concluded our list of physical vertex operators leading to (anti-)chiral operators in spacetime.

B. Cyclic orbifolds

In this appendix we collect some results about cyclic orbifolds which are relevant to the discussion of CFTs on symmetric products in section 5.

Assume that we take a general CFT $\mathcal{M}$ with some symmetry algebra (which for convenience we will consider to be of rank 1 and designate its Cartan generator by J), then the generalized partition function is defined to be:

$$Z(\tau,z,\bar\tau,\bar z) = q^{-c/24}\bar q^{-\bar c/24}\,\mathrm{Tr}\, q^{L_0}\bar q^{\bar L_0} x^J \bar x^{\bar J} = \sum_{k,l} N_{kl}\chi_k(\tau,z)\bar\chi_k(\bar\tau,\bar z)\,. \tag{B.1}$$

The generalized characters are defined as:

$$\chi_k(\tau,z) = \mathrm{Tr}_{\mathcal{H}_k}\, e^{2\pi i z J} e^{2\pi i\tau(L_0-c/24)}\,. \tag{B.2}$$

χ_k is the character corresponding to a primary field ψ_k where the trace is taken on the Virasoro algebra representation space $\mathcal{H}_k$ (i.e. the descendants) and $q = e^{2\pi i\tau}$,

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$x = e^{2\pi i z}$. We now look at the cyclic orbifold $\mathcal{M}^N/Z_N$ for N prime. In [22, 23] the modular invariant partition function of such a cyclic orbifold model is shown to be:

$$\mathcal{Z}(\tau, z) = \frac{Z^N(\tau, z)}{N} + \frac{(N-1)}{N}\left[Z(N\tau, Nz) + \sum_{n=0}^{N-1} Z\left(\frac{\tau+n}{N}, z\right)\right]. \tag{B.3}$$

This is obtained by taking the Z_N invariant projection on the $\mathcal{M}^N$ partition function and then adding the terms required by modular invariance. The first two terms correspond to the untwisted sector, and the last ones to the twisted sectors. Note that the $N-1$ factor in front of the twisted part appears because we have $N-1$ identical twisted sectors in a Z_N orbifold (when N is prime). From this very general formula one can already extract the vacuum energies and J charges of states in the twisted sectors [22, 23]. To compute the vacuum energies in the twisted sectors note that the modular parameter in the twisted part of the partition function has become $(\tau+n)/N$. Plugging this into the definition of the character and rearranging to get an expression of the form

$$\chi(\tau) = q^{-\tilde{c}/24}\,\mathrm{Tr}_{\mathcal{H}}\, q^{\tilde{L}_0}, \tag{B.4}$$

where $\tilde{c} = Nc$ and $\tilde{L}_0$ are the central charge and energy of the twisted theory in the appropriate twisted sector, we get:

$$\begin{aligned} \chi_k\left(\frac{\tau+n}{N}\right) &= e^{2\pi i\left(\frac{\tau+n}{N}\right)\left(-\frac{c}{24}\right)}\,\mathrm{Tr}\, e^{2\pi i\left(\frac{\tau+n}{N}\right)L_0} \\ &= q^{-\frac{c}{24N}}\,\mathrm{Tr}\, q^{\frac{L_0}{N}} e^{2\pi i\left(\frac{n}{N}\right)\left(L_0-\frac{c}{24}\right)} \\ &= q^{-\frac{Nc}{24}}\,\mathrm{Tr}\, q^{\frac{L_0}{N}+\frac{c}{24}\left(N-\frac{1}{N}\right)} e^{2\pi i\left(\frac{n}{N}\right)\left(L_0-\frac{c}{24}\right)}. \end{aligned} \tag{B.5}$$

The term $1/N\sum_{n=0}^{N-1}\exp[2\pi i(n/N)(L_0 - c/24)]$ together with the anti-holomorphic contribution serves as a projection operator onto $h - \bar{h} = 0$ (mod N). We can thus identify the weight of states generated by an operator of the original theory (with weight h) in the Z_N twisted sectors of the orbifold to be:

$$h^N = \frac{h+m}{N} + \frac{c}{24}\frac{N^2-1}{N}, \tag{B.6}$$

where $m = 0, 1, \ldots, N-1$ and we denote by N quantities in the twisted sectors. This means that every primary field and its first $N-1$ descendants in the original theory $\mathcal{M}$ give rise to a primary field in the Z_N orbifold theory with weight given by (B.6). In particular this gives the vacuum energies in the twisted sectors as $(c/24)(N^2-1)/N$. Every state of a given weight in the twisted sector has multiplicity $N-1$.

Another conclusion arises from examining the second argument of the twisted part of the orbifold partition function in (B.3) which is $Z(\tau + n/N, z)$. This shows that the symmetry charges (J in our case) carried by the operators in the Z_N twisted sectors are identical to those in the untwisted sector [23], we thus write:

$$R^N = R. \tag{B.7}$$

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We now generalize the expressions for non-prime N. This is derived in essentially the same way but now we should remember that there are elements of the group Z_N that have degree $r \leq N$ and so complete a full cycle in r steps, i.e. $b^r = 1$. In the above derivation the fact that N was prime was used to make sure that all the elements are of degree N and thus all twisted sectors are equivalent. For non-prime N there are elements in Z_N whose degree r is a divisor of N. The sectors twisted by these elements and their powers will lead to terms in the partition function which are similar to the ones in (B.3) but with N replaced by r, and each one to the power of N/r since this is the number of disconnected cycles. Therefore we should change (B.3) to:

$$\mathcal{Z}(\tau, z) = \frac{Z^N(\tau, z)}{N} + \frac{1}{N}\sum_{m=1}^{N-1}\left[Z^{\frac{N}{r_m}}(r_m\tau, r_m z) + \sum_{n=0}^{r_m-1} Z^{\frac{N}{r_m}}\left(\frac{\tau+n}{r_m}, z\right)\right] + \cdots\,, \tag{B.8}$$

where r_m is the degree of a^m, with a a generator of Z_N $(r_1 = N)$. We see that for N prime the degree of all the elements $r_m = N$ is independent of m and we recover (B.3) . The terms omitted from this formula describe sectors where the twist in the timelike direction of the torus corresponds to an element of Z_N with higher degree then the element corresponding to the twist in the spacelike direction. The outcome of this discussion is that for any N, there are twisted states obeying (B.6) and (B.7), however their multiplicity is less than $N-1$ if N is not prime.

Note that although in the $\mathcal{M}^N/Z_N$ orbifold the Z_N twist fields come with non-trivial multiplicities, when one considers the full $\mathcal{M}^p/S_p$ orbifold these same twist fields actually come with multiplicity one. The reason is that in the S_p group, a Z_N cycle is one conjugacy class and thus gives rise to only one twisted sector.

The explicit forms of the Z_N partition function (B.3) and (B.8) allows us to heuristically distinguish between the single particle and multi-particle spectrum, by simply noting the power to which the partition function of $\mathcal{M}$ is raised. The terms related to twisted sectors of maximal degree N appear to unit power and thus correspond to single particle states, while for instance the first term Z^N indicates that in the untwisted sector all the states that are not in the diagonal $\mathcal{M}$ arise from multiplying operators in the various copies, and thus are identified with multi-particle states.

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